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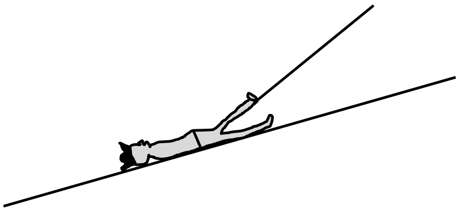
You must use <u>vector and calculus methods</u> throughout this paper. Use of the equations of motion under constant acceleration are not allowed. Assume g = 9.8 m/s^2.

1. **Draw labelled force diagrams** to show the forces involved in each of the following situations as **forces acting on a single point**. Indicate the relative sizes of the forces by the length of the vector arrows.
   a) a golf ball being hit (during the impact)
   b) a car accelerating on a horizontal surface
   c) a dead pirate being dragged up a sloping deck by a rope as shown below



2. The displacement of a parrot at time t (sec) is given by $\boldsymbol{r} = 10t\boldsymbol{i} + (5t - {}^1\!/_2 t^2)\boldsymbol{j}$.
   Find the velocity of the parrot when $t = 6$.

3. **Draw a graph** of the path of the parrot in the previous question from $t = 0$ to $t = 12$. Include labels and scales on the axes. **Express this curve** in the form $y = f(x)$.

4. A 50 kg cannon is standing on a slippery deck sloping 20º. It is held in place by a rope parallel to the deck. Assuming there is no friction, **find the tension in the rope** and **the magnitude of the normal reaction of the deck on the cannon**.

5. A longboat needs to reach an island due north of its present position. It can do 6 knots through the water. But there is a 2 knot current towards the southeast. **In what direction should the boat head in order to arrive at the island?**

6. Bluebeard the pirate wanted a flag that has a rhombus with bones making the two diagonals. Furthermore, he wanted the bones not to be perpendicular. But, however he drew the rhombus, the bones always seemed to be at right angles. **Use vectors to prove to Bluebeard that what he wanted is not possible.** [A rhombus is a parallelogram with four equal sides.]

7. A rocket was fired from a ship and moved in a vertical east-west plane. $\boldsymbol{i}$ is the unit vector towards the east; $\boldsymbol{j}$ is the unit vector vertically upwards. At time $t = 0$ (t is in seconds) the rocket was launched from the point with position vector $5\boldsymbol{j}$ and moving at 10 m/s vertically upwards. From then on, it accelerated with acceleration equal to $5t\boldsymbol{i} - 3\boldsymbol{j}$. **Find its position when $t = 5$.**

You must use <u>vector and calculus methods</u> throughout this paper. Use of the equations of motion under constant acceleration are not allowed. Assume g = 9.8 m/s^2.

1. A cannon ball is fired from 3 m above the sea at a speed of 35 m/s and at 40º above the horizontal. Will it hit a 20 m high pirate ship 100 m away in the direction of travel.
   State any assumptions you make in reaching your conclusion.

2. a) Jim Hawkins was driving a car with a mass of 1500kg in an easterly direction at 70 km/h. It collided with a 4WD with a mass of 2150kg which was travelling southeast. After the collision, the two vehicles remained attached to each other and travelled off in a direction of 25° S of E. **Find the speed of the 4WD before the collision and determine if it was exceeding the 80km/h speed limit.**

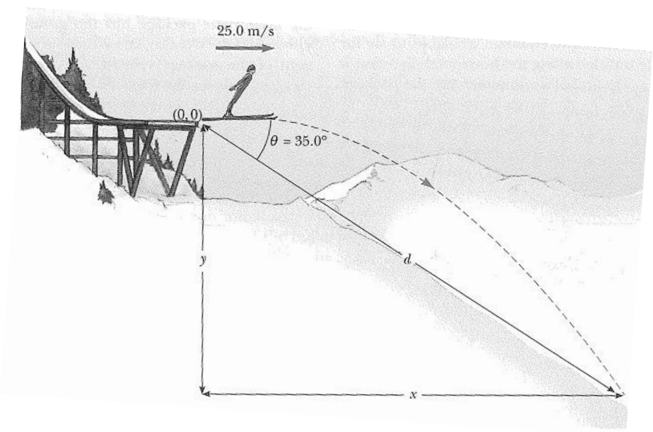
   b) With a bit of extra information, the model you used could be refined to give a more accurate result. **What could the information be and how would it be used to improve the accuracy?**

3. A plank sticks out 2.4 m horizontally from the side of a ship. A pirate is standing on the end of the plank. The plank is slowly tilted downwards. The coefficient of friction between the pirate's boots and the plank is about 0.3. The captain works out that if the outer end of the plank is lowered by 1.4 m, the pirate will slip off into the sea where he will be eaten by sharks.

   Determine the validity of the captain's reasoning.

   What assumptions must he make?

4. Mr Sparrow, the ski jumper, leaves a ski track moving in the horizontal direction with a speed of 25 m/s (see below). The snow he lands on slopes down at 35° from the point where he leaves the track. **Use this information to generate a mathematical model of his flight. Then use the model to determine how far down the slope he lands (the distance *d* on the diagram).**



Q5 on the next page.

5. The diagram represents part of an island. Two pirates came to the island to bury some treasure.

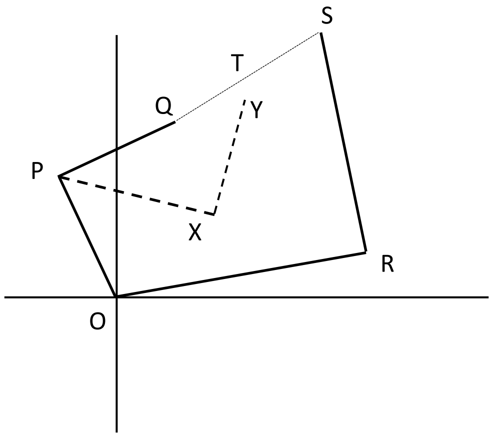
Both started at a tree at O.

Pirate A walked to a second tree at P, then turned 90º right and walked the same distance again to Q.

Pirate B also started at the tree at O, but walked to a third tree at R, then turned 90º left and walked the same distance again to S.

They then buried the treasure at T, half way between Q and S.

Pirate C later came to find the treasure. She started at P, walked to point X, half way between P and R, then turned left and walked the same distance again to point Y.



Pirate C found the treasure because T and Y are in fact the same point. Below is a proof of this, though some parts of the proof have been eaten by insects.

Let the diagram represent the complex plane with O at the origin and the real and imaginary axes shown. Let the points P and R be represented by the complex numbers u and v respectively.

Then $Q = u - iu$ *and* $S =$

$T =$

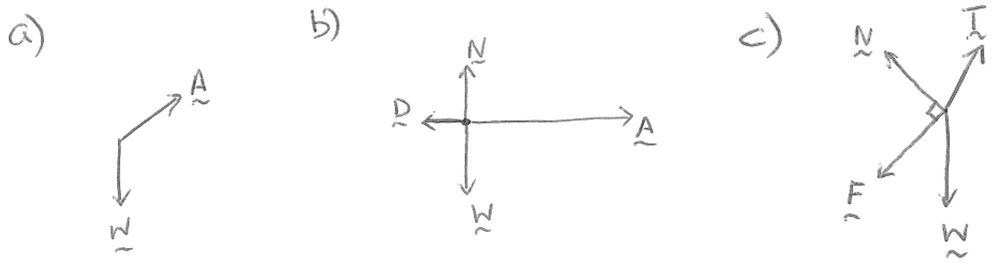
$X=$

$Y =$ $\qquad\qquad\qquad$ $=$ $\qquad\qquad\qquad$ $= T$

Your job is to use ideas from vectors and complex numbers to **rewrite the last four lines** with the missing parts replaced. No other working is required.

Year 11 Maths C Term 4 KP Solutions

1.



2.

$$\boldsymbol{r}(t) = 10t\boldsymbol{i} - \left(5t - \frac{t^3}{3}\right)\boldsymbol{j}$$

$$\boldsymbol{v}(t) = \dot{\boldsymbol{r}}$$

$$= 10\boldsymbol{i} + (5 - t^2)\boldsymbol{j}$$

$$\boldsymbol{v}(5) = 10\boldsymbol{i} - 20\boldsymbol{j}\ m/s^2$$

3.

$$x = 10t \ and \ y = 5t - \frac{t^3}{3}$$

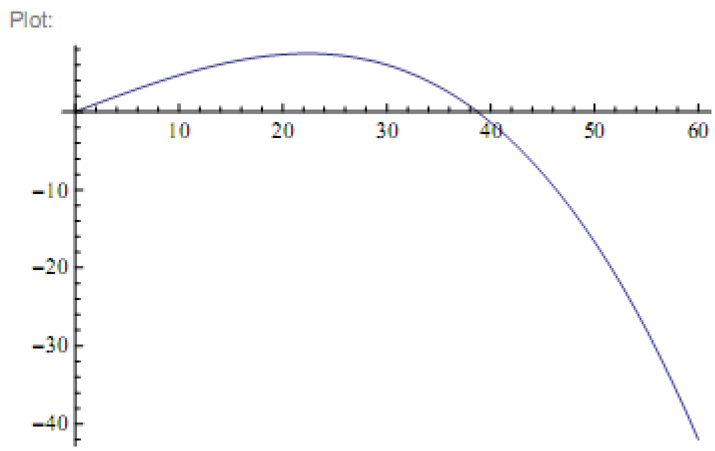
$$Then \ t = \frac{x}{10} \ and \ sub \ into \ y$$

$$y = 5 \times \frac{x}{10} - \frac{\left(\frac{x}{10}\right)^3}{3}$$

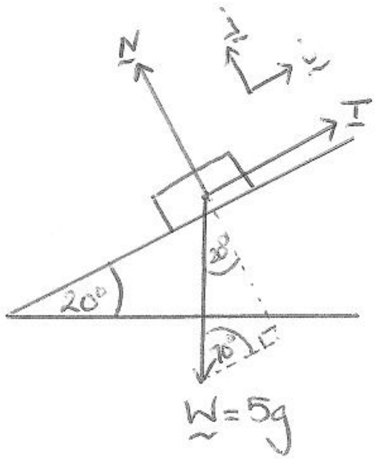
$$y = \frac{x}{2} - \frac{x^3}{3000}$$

$$Therefore \ the \ Cartesian \ equation \ of \ the \ trajectory \ of \ the \ particle \ is$$

$$y = \frac{x}{2} - \frac{x^3}{3000}$$



4.



$\boldsymbol{W} = -5g\cos70\boldsymbol{i} - 5g\sin70\boldsymbol{j}$

$\boldsymbol{T} = T\boldsymbol{i} \quad and \quad \boldsymbol{N} = N\boldsymbol{j}$

$\boldsymbol{T} + \boldsymbol{W} + \boldsymbol{N} = 0$

$\therefore T\boldsymbol{i} - 5g\cos70\boldsymbol{i} = 0$

$T = 5g\cos70$

$T = 16.76N$

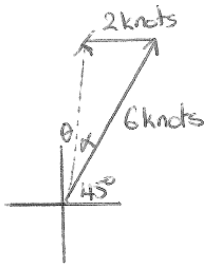
$\therefore N\boldsymbol{j} - 5g\sin70\boldsymbol{j} = 0$

$N = 5g\sin70$

$N = 46.04N$

Hence the Tension and Normal reactive force are $16.76N$ *and* $46.04N$ *respectively*

5.



$Boat = 6\cos45\boldsymbol{i} + 6\sin45\boldsymbol{j}$

$Boat = 4.24\boldsymbol{i} + 4.24\boldsymbol{j}$

$\boldsymbol{v}_b = v_{b\,rel\,c} + \boldsymbol{v}_c$

$\boldsymbol{v}_b = 4.24\boldsymbol{i} + 4.24\boldsymbol{j} - 2\boldsymbol{i}$

$\boldsymbol{v}_b = 2.24\boldsymbol{i} + 4.24\boldsymbol{j}$

$\therefore |\boldsymbol{v}_b| = \sqrt{(2.24^2 + 4.24^2)}$

$|\boldsymbol{v}_b| = 4.8\ knots$

$$\frac{2}{\sin\emptyset} = \frac{4.8}{\sin 45}$$

$$\emptyset = \sin^{-1}\left(\frac{2\sin45}{4.8}\right)$$

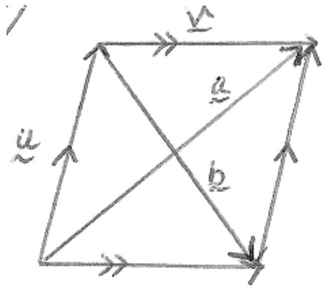
$\emptyset = 17.13°$

$\therefore \theta = 45 - 17.13$

$\theta = 27.86°$ *East of North or* $27°51'$

The boat will travel at 4.8 *knots at an angle of* $27.86°$ *East of North*

6.



$\boldsymbol{a} = \boldsymbol{u} + \boldsymbol{v}$

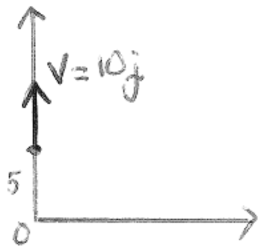
$\boldsymbol{b} = \boldsymbol{v} - \boldsymbol{u}$

$$\boldsymbol{a} \cdot \boldsymbol{b} = (\boldsymbol{u} + \boldsymbol{v})(\boldsymbol{v} - \boldsymbol{u})$$
$$= \boldsymbol{v}^2 - \boldsymbol{u}^2$$
$$= \boldsymbol{v}.\boldsymbol{v} - \boldsymbol{u}.\boldsymbol{u}$$
$$= |\boldsymbol{v}|^2 - |u|^2 \qquad and\ |\boldsymbol{v}| = |\boldsymbol{u}|$$

$Then\ \boldsymbol{a} \cdot \boldsymbol{b} = 0\ and\ hence\ are\ perpendicular$

$Therefore\ the\ diagonals\ \boldsymbol{a}\ and\ \boldsymbol{b}\ intersect\ at\ right\ angles$

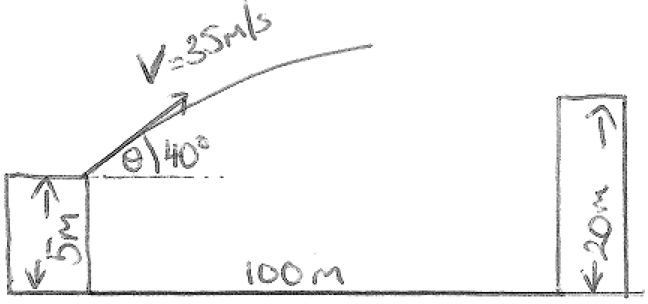
7.



$$\boldsymbol{a} = 5t\boldsymbol{i} - 3\boldsymbol{j}$$
$$\boldsymbol{v} = \int \boldsymbol{a}\, dt$$
$$\boldsymbol{v} = \frac{5}{2}t^2\boldsymbol{i} - 3t\boldsymbol{j} + \boldsymbol{c_1}$$
$$@\ t = 0, \boldsymbol{v} = 10\boldsymbol{j}\ and\ \boldsymbol{v}(0) = \boldsymbol{c_1}, \therefore \boldsymbol{c_1} = 10\boldsymbol{j}$$
$$so\ \boldsymbol{v} = \frac{5}{2}t^2\boldsymbol{i} + (10 - 3t)\boldsymbol{j}$$
$$then\ \boldsymbol{r} = \int \boldsymbol{v}\, dt$$
$$\boldsymbol{r} = \frac{5}{6}t^3\boldsymbol{i} + \left(10t - \frac{3}{2}t^2\right)\boldsymbol{j} + \boldsymbol{c_2}$$
$$@\ t = 0, \boldsymbol{r} = 5\boldsymbol{j}\ and\ \boldsymbol{r}(0) = \boldsymbol{c_2}, \therefore \boldsymbol{c_2} = \boldsymbol{5j}$$
$$so\ \boldsymbol{r} = \frac{5}{6}t^3\boldsymbol{i} + \left(10t - \frac{3}{2}t^2 + 5\right)\boldsymbol{j}$$
$$Then\ \boldsymbol{r}(5) = \frac{5}{6}(5)^3\boldsymbol{i} + \left(10 \times 5 - \frac{3}{2}(5)^2 + 5\right)\boldsymbol{j}$$
$$\boldsymbol{r}(5) = 104\frac{1}{6}\boldsymbol{i} + 17.5j$$

Therefore the position of the rocket at $t = 5$ is $104\frac{1}{6}\boldsymbol{i} + 17.5j$

8.



$$V = v_x\boldsymbol{i} + v_y\boldsymbol{j}$$
$$V = 35cos40\boldsymbol{i} + 35sin40\boldsymbol{j}$$
$$V = 26.8\boldsymbol{i} + 22.5\boldsymbol{j}$$

$$Time\ to\ complete\ 100m = 100\ \div 26.8$$
$$Time\ to\ complete\ 100m = 3.73secs$$

$$\boldsymbol{a} = -g\boldsymbol{j}$$
$$\boldsymbol{v} = \int -g\boldsymbol{j}\ dt$$
$$\boldsymbol{v} = -gt\boldsymbol{j} + \boldsymbol{c}_1$$
$$when\ t = 0, \boldsymbol{v} = 0\ and\ \boldsymbol{c}_1 = 26.8\boldsymbol{i} + 22.5\boldsymbol{j}$$
$$\therefore \boldsymbol{v} = 26.8\boldsymbol{i} + (22.5 - gt)\boldsymbol{j}$$
$$\boldsymbol{r} = \int 26.8\boldsymbol{i} + (22.5 - gt)\boldsymbol{j}\ dt$$
$$\boldsymbol{r} = 26.8t\boldsymbol{i} + \left(22.5t - \frac{gt^2}{2}\right)\boldsymbol{j} + \boldsymbol{c}_2$$
$$when\ t = 0, \quad r(0) = 0\boldsymbol{i} + 5\boldsymbol{j}\ and\ hence\ \boldsymbol{c}_2 = 0\boldsymbol{i} + 5\boldsymbol{j}$$
$$\therefore \boldsymbol{r} = 26.8t\boldsymbol{i} + \left(22.5t - \frac{gt^2}{2} + 5\right)\boldsymbol{j}$$

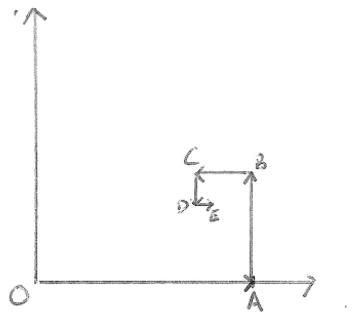
$Find\ the\ height\ at\ 3.72s\ by\ subbing\ into\ r_y$

$r_y(3.73) = 20.75m$

$Therefore\ the\ cannon\ ball\ will\ jut\ pass\ above\ the\ ship\ and\ not\ hit\ it$

Year 11 Maths C Term 4 MPS Solutions

1.



Point A (4,0)
Point B (4,2)
Point C (3,2)
Point D (3,1.5)
Point E (3.25,1.5)

Amount of movement in x values: 4, -1, ¼,....
Amount of movement in y values: 2, -½,.....

Since pattern continues to infinity find S_∞ for geometric progression $S_\infty = \frac{a}{1-r}$

$$for\ x\ values\ S_\infty = \frac{4}{1-\frac{-1}{4}}$$
$$S_\infty = \frac{16}{5}$$
$$for\ y\ values\ S_\infty = \frac{2}{1-\frac{-1}{4}}$$
$$S_\infty = \frac{8}{5}$$

Therefore position vector after infinite iterations is $P = \frac{16}{5}\boldsymbol{i} + \frac{8}{5}\boldsymbol{j}$

Distance from origin:

$$|P| = \sqrt{\left(\frac{16}{5}\right)^2 + \left(\frac{8}{5}\right)^2}$$
$$|P| = 3.58\ km$$

Direction from origin:

$$\theta = tan^{-1}\left(\frac{\frac{8}{5}}{\frac{16}{5}}\right)$$
$$\theta = 26.57°\ or\ 26°33'$$

Jack after infinite iterations is 3.58km from the origin in a direction of $N26°33'E$

2.

$$Car\ A: m = 1500kg, v = 70km/h\ \approx 19.4m/s\ travelling\ East$$
$$CarB: m = 2150kg, v = ?\ travelling\ SE$$

Before collision

$$Momentum\ of\ Car\ A$$
$$= 1500(19.4\boldsymbol{i} + 0\boldsymbol{j})$$
$$= 29100\boldsymbol{i}$$
$$Momentum\ of\ Car\ B$$
$$= 2150(vcos45\boldsymbol{i} - vsin45\boldsymbol{j})$$
$$= 2150vcos45\boldsymbol{i} - 2150vsin45\boldsymbol{j}$$

In a collision, momentum is conserved and momentum before = momentum after

After collision, cars are together travelling $E25°S$

$$\therefore (29100\boldsymbol{i} + 2150vcos45)\boldsymbol{i} - 2150vsin45\boldsymbol{j} = Tcos25\boldsymbol{i} - Tsin25\boldsymbol{j}$$
$$where\ T\ is\ the\ momentum\ of\ both\ vehicles\ after\ the\ collision$$

$$Equate\ horizontal\ and\ vertical\ components|$$
$$Horizontal:\ \ 29100 + 2150vcos45 = Tcos25 \qquad (1)$$
$$Vertical:\ \ 2150vsin45 = Tsin25 \qquad (2)$$
$$Note: sin45 = cos45\ \therefore (2) becomes\ 2150vcos45 = Tsin25 \qquad (3)$$
$$Sub(3) into\ (1)$$
$$29100 + Tsin25 = Tcos25$$
$$rearranging$$
$$T = \frac{29100}{cos25 - sin25}$$
$$T = 60163\ Ns$$

$$sub\ T = 60163\ into\ (2)\ to\ find\ v$$
$$2150vsin45 = 60163sin25$$
$$v = \frac{60163sin25}{2150sin45}$$
$$v \approx 16.72m/s$$
$$v \approx 60.2km/h$$

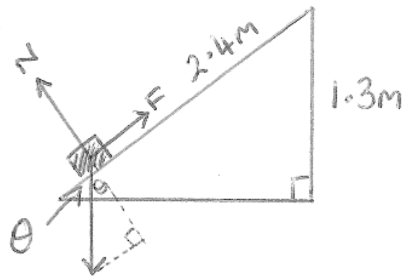
Therefore Car B was not speeding

3.

$Friction = \mu N$

$\boldsymbol{W} = W_x\boldsymbol{i} + W_y\boldsymbol{j}$
$\boldsymbol{W} = -mgsin\theta\boldsymbol{i} - mgcos\theta\boldsymbol{j}$

$N = -W_y \text{ and } F = -W_x$
$N = mgcos\theta \text{ and } F = mgsin\theta$

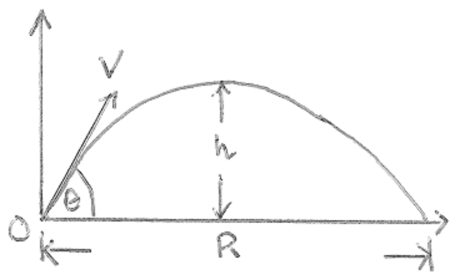


$Friction = \mu \times mgcos\theta \text{ and } F = mgsin\theta$
$\therefore \mu \times mgcos\theta = mgsin\theta$
$$\mu = \frac{mgsin\theta}{mgcos\theta}$$
$\mu = tan\theta$

$$\mu = \tan\left(\sin^{-1}\left(\frac{1.3}{2.4}\right)\right)$$
$\mu = 0.644$

4.

$\boldsymbol{a} = -g\boldsymbol{j}$
$\boldsymbol{v} = -gt\boldsymbol{j} + \boldsymbol{c_1}$
$@\, t = 0 \quad \boldsymbol{V} = v_x\boldsymbol{i} + v_y\boldsymbol{j}$
$\therefore \boldsymbol{V}(0) = Vcos\theta i + Vsin\theta j$
$\text{and since } @\, t = 0 \quad \boldsymbol{v} = \boldsymbol{c_1}$
$\therefore \boldsymbol{v} = Vcos\theta\boldsymbol{i} + (Vsin\theta - gt)\boldsymbol{j}$
$$\boldsymbol{r} = Vtcos\theta\boldsymbol{i} + \left(Vtsin\theta - \frac{1}{2}gt^2\right)\boldsymbol{j} + \boldsymbol{c_2}$$
$\boldsymbol{c_2} = 0$ *as the origin is the point of projection*



Range occurs at 2t as the path is parabolic and hence symmetrical about the maximum height and where t is the time at which the maximum height occurs

For maximum height $v_y = 0$
$$0 = Vsin\theta - gt$$
$$t = \frac{Vsin\theta}{g} \text{ is the time at which the max height occurs}$$

$\therefore$ max *range occurs at* $\frac{2Vsin\theta}{g}$ *, sub value into* r_x *and let it be R*
$$R = \frac{2V^2sin\theta cos\theta}{g}$$
since $\sin 2\theta = 2sin\theta cos\theta$
$$R = \frac{V^2sin2\theta}{g} \text{ is the expression for the range}$$

To find the optimal angle of projection need to maximise R and hence maximises $\sin 2\theta$

$$Maximum \text{ of } sin2\theta = 1$$

$$2\theta = 90°$$
$$\therefore \theta = 45° \; produces\ the\ maximum\ range\ for\ any\ initial\ velocity\ V$$

Assumptions that are made are that:
1. That air resistance is negligible
2. The projectile is lands within the same plane
3. The projectile is fired from the ground and lands at the same height at which it is fired from i.e. 0m

Note: Answer for expression may vary dependent on initial position, but this must be stated in the assumptions.

5. Diagram on exam paper

$\boldsymbol{a} = -g\boldsymbol{j}$
$\boldsymbol{v} = -gt\boldsymbol{j} + \boldsymbol{c_1}$
$@\, t = 0 \quad V = v_x\boldsymbol{i} + v_y\boldsymbol{j}$
$\therefore\ V = 25\boldsymbol{i}$
$and\ since\ @\, t = 0 \quad \boldsymbol{v} = \boldsymbol{c_1}$
$\therefore \boldsymbol{v} = 25\boldsymbol{i} - gt\boldsymbol{j}$
$\boldsymbol{r} = 25t\boldsymbol{i} - \frac{1}{2}gt^2\boldsymbol{j} + \boldsymbol{c_2}\ but\ c_2 = 0\ as\ the\ skier\ launches\ from\ the\ origin$
$so\ \boldsymbol{r} = 25t\boldsymbol{i} - \frac{1}{2}gt^2\boldsymbol{j}$

$@\, t = 0 \ \ V = 25\boldsymbol{i}$
$\sin 35 = \frac{y}{d}\ and\ \cos 35 = \frac{x}{d}$
$y = dsin\ 35\ and\ x = dcos35$

$Then\ equating\ x\ and\ y\ with\ r_x\ and\ r_y$
$\therefore dcos\ 35 = 25t$
$t = \frac{dcos35}{25}$
$sub\ t\ into\ \frac{-1}{2}gt^2 = dsin\ 35$
$-\frac{1gd^2cos^2 35}{1250} = dsin35$
$-0.00526d^2 = 0.57357d$
$0 = 0.00526d^2 + 0.5735d$
$Solve\ via\ Graphics\ Calculator$
$d = 0m\ or\ d = 109.03m$

$Therefore\ the\ straight\ line\ distance\ travelled\ by\ the\ skier\ is\ 109.03m$