Matrices

M1. If $\boldsymbol{A} = \begin{pmatrix} 2 & 2 \\ -1 & 0 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ -3 & -2 \end{pmatrix}$, without using a calculator, find:

(a) $\boldsymbol{A} + \boldsymbol{B}$ (b) $\boldsymbol{A} - \boldsymbol{B}$ (c) $5\boldsymbol{A}$ (d) $\boldsymbol{AB}$ (e) $\boldsymbol{BA}$ (f) $\boldsymbol{A}^2$
(g) $\boldsymbol{A}^{\mathrm{T}}$ (h) $|\boldsymbol{A}|$ (i) $\boldsymbol{A}^{-1}$ (j) $\boldsymbol{B}\boldsymbol{B}^{-1}$ (k) $\boldsymbol{B}^{-1}\boldsymbol{B}$

M2. If $\boldsymbol{A} = \begin{pmatrix} 1 & 0 & -4 \\ -1 & 2 & 4 \\ 2 & 1 & 0 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 2 & -1 & -1 \\ 1 & 2 & -2 \\ 0 & 1 & 1 \end{pmatrix}$, using a calculator where necessary, find:

(a) $\boldsymbol{A} + \boldsymbol{B}$ (b) $\boldsymbol{A} - \boldsymbol{B}$ (c) $-\boldsymbol{A}$ (d) $\boldsymbol{AB}$ (e) $\boldsymbol{BA}$ (f) $\boldsymbol{A}^2$
(g) $\boldsymbol{A}^{\mathrm{T}}$ (h) $|\boldsymbol{A}|$ (i) $\boldsymbol{A}^{-1}$ (j) $\boldsymbol{B}\boldsymbol{B}^{-1}$ (k) $\boldsymbol{B}^{-1}\boldsymbol{B}$

M3. Use the 2×2 matrix method with the encryption matrix $\begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$ to encode:

(a) Hi there (b) What you doing

M4. The following messages were encoded using the 2×2 matrix method with the encryption matrix $\begin{pmatrix} 2 & -1 \\ 1 & -2 \end{pmatrix}$. Decode them.

(a) 39602225 (b) 894668187338391783406215

M5. Write a matrix for each of the following transformations of the x-y plane:

(a) a dilation by a factor of 4 from the x-axis
(b) a dilation by a factor of ½ from the y-axis
(c) a dilation by a factor of 2 from the origin
(d) a reflection in the x-axis
(e) a reflection in the y-axis
(f) a reflection in the line $y = -x$
(g) a rotation of 180° about the origin
(h) a rotation of –90° about the origin

M6. Write a single matrix for each of the following sequences of transformations of the x-y plane:

(a) a dilation by a factor of 2 from the origin followed by a reflection in the y-axis
(b) a rotation of 90° about the origin followed by a reflection in the line $y = x$
(c) a rotation of 40° about the origin followed by a rotation of –130° about the origin
(d) a rotation of 60° about the origin followed by a reflection in the x-axis, then a rotation of –30° about the origin

Vectors

V1. If $\boldsymbol{A} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, find
(a) the scalar product $\boldsymbol{A}.\boldsymbol{B}$
(b) the angle between the vectors

V2. If $\boldsymbol{A} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}$, find
(a) the dot product $\boldsymbol{A}.\boldsymbol{B}$
(b) the angle between the vectors

V3. Express the polar vector $r = 6$ $\theta = 125°$ $\varphi = -40°$ in terms of unit vectors $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$.

V4. Express $-4\boldsymbol{i} + 2\boldsymbol{j} + 5\boldsymbol{k}$ in polar form.

V5. If Brisbane is at 27°S 153°E and Bangkok is at 14°N 101°E, how far is it from Brisbane to Bangkok by plane? [Assume that the circumference of the Earth is 40 000 km.]

V6. If Brisbane is at 27°S 153°E and New York is at 41°N 74°W, how far is it from Brisbane to New York by plane? [Assume that the circumference of the Earth is 40 000 km.]

V7. Pogo is flying the shortest air route from Hobart (44°S 148°E) to Buenos Aires (35°S 58°W). What are his latitude and longitude when he is half way there?

V8. If $\boldsymbol{A} = 2\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\boldsymbol{B} = -\boldsymbol{i} + 2\boldsymbol{k}$, find (a) $\boldsymbol{A}\times\boldsymbol{B}$ (b) $\boldsymbol{B}\times\boldsymbol{A}$

V9. If $\boldsymbol{A} = \begin{pmatrix} 4 \\ -11 \\ 7.5 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 12 \\ 5 \\ -6 \end{pmatrix}$, find (a) $\boldsymbol{A}\times\boldsymbol{B}$ (b) $\boldsymbol{B}\times\boldsymbol{A}$

V10. If $\boldsymbol{A} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$, find the scalar triple product $\boldsymbol{A}.\boldsymbol{B}\times\boldsymbol{C}$

V11. A parallelepiped has one corner at (2, 1, –1) and edges going from that corner to (4, 2, 0) , (3, 4, –2) and (5, 3, 5) . Find its volume.

V12. A parallelepiped has one corner at (0, 4, –1) and edges going from that corner to (2, 4, 1) , (3, 3, –2) and (4, 0, –1) . Find its volume.

Answers

M1. (a) $\begin{pmatrix} 3 & 2 \\ -4 & -2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$ (c) $\begin{pmatrix} 10 & 10 \\ -5 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} -4 & -4 \\ -1 & 0 \end{pmatrix}$

(e) $\begin{pmatrix} 2 & 2 \\ -4 & -6 \end{pmatrix}$ (f) $\begin{pmatrix} 2 & 4 \\ -2 & -2 \end{pmatrix}$ (g) $\begin{pmatrix} 2 & -1 \\ 2 & 0 \end{pmatrix}$ (h) 2

(i) $\begin{pmatrix} 0 & -1 \\ 0.5 & 1 \end{pmatrix}$ (j) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (k) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

M2. (a) $\begin{pmatrix} 3 & -1 & -5 \\ 0 & 4 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 1 & -3 \\ -2 & 0 & 6 \\ 2 & 0 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & 0 & 4 \\ 1 & -2 & -4 \\ -2 & -1 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & -5 & -5 \\ 0 & 9 & 1 \\ 5 & 0 & -4 \end{pmatrix}$

(e) $\begin{pmatrix} 1 & -3 & -12 \\ -5 & 2 & 4 \\ 1 & 3 & 4 \end{pmatrix}$ (f) $\begin{pmatrix} -7 & -4 & -4 \\ 5 & 8 & 12 \\ 1 & 2 & -4 \end{pmatrix}$ (g) $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & 1 \\ -4 & 4 & 0 \end{pmatrix}$ (h) 16

(i) $\begin{pmatrix} -0.25 & -0.25 & -0.5 \\ 0.5 & 0.5 & 0 \\ -0.312 & -0.062 & -0.125 \end{pmatrix}$ (j) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (k) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

M3. (a) 4660184355604516 (b) 954671187976335766613745

M4. (a) Boot (b) That's correct

M5. (a) $\begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(e) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ (f) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ (g) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ (h) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

M6. (a) $\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

V1. (a) –1 (b) 98°

V2. (a) 2 (b) 80°

V3. $-2.64\boldsymbol{i} + 3.76\boldsymbol{j} - 3.86\boldsymbol{k}$

V4. $r = 6.71$ $\theta = 153.4°$ $\varphi = 48.2°$

V5. 7224 km

V6. 15 461 km

V7. 69°S 129°W

V8. (a) $-4\boldsymbol{i} - 7\boldsymbol{j} - 2\boldsymbol{k}$ (b) $4\boldsymbol{i} + 7\boldsymbol{j} + 2\boldsymbol{k}$

V9. (a) $\begin{pmatrix} 28.5 \\ -114 \\ 152 \end{pmatrix}$ (b) $\begin{pmatrix} -28.5 \\ 114 \\ -152 \end{pmatrix}$

V10. 42

V11. 24

V12. 24