

Normal distributions

Use normal distribution mathematics for these questions.

1. You toss a coin 200 times.
 - a. Find the mean number of heads and the standard deviation. [100, 7.07]
 - b. Find the probability of getting between 105 and 110 (inclusive) heads. [0.161]*
 - c. Find the probability of getting more than 96 heads. [0.714]*
 - d. Find the probability of getting fewer than 90 heads. [0.079]*

2. You roll a die 600 times
 - e. There is an approximately 10% probability of getting more than x sixes. Find x . [112]*
 - f. There is an approximately 3.5% probability of getting less than x sixes. Find x . [83]*
 - g. There is a roughly 60% probability of getting within x sixes of 100. Find x . [8]*
 - h. There is a roughly 77% chance of getting between 80 and x (inclusive) heads. Find x . [107]*

3. Adults' heights are normally distributed with mean 170 cm and standard deviation 8 cm
 - a. What percentage of adults are taller than 180 cm? [10.6%]
 - b. What percentage of adults are shorter than 155 cm? [3.0%]
 - c. What percentage of adults are between 165 and 178 cm tall [57.5%]
 - d. How tall would you have to be to be in the tallest 10% of the population? [180.2 cm]
 - e. How tall would you have to be to be in the shortest 2% of the population? [153.6 cm]
 - f. What is the height range of the middle 60% of the population? [163.2 cm – 176.7 cm]
 - g. 40% of the population have heights between 165 cm and what? [173.4 cm]

4. The masses of M&Ms are normally distributed with mean 360 mg and sd 20 mg.
 - a. What percentage of M&Ms weigh more than 400 mg? [2.28%]
 - b. What percentage weigh less than 300 mg? [0.135%]
 - c. If only those between 330 and 410 mg are accepted for putting into packets, out of a batch of 50 000, how many would you expect to be rejected? [3651]
 - d. If the manufacturer wanted instead just to reject the smallest 1%, what mass would the cut-off need to be? [313.5 mg]
 - e. A 400 mg M&M is 2 standard deviations above the mean. How many standard deviations above the mean is a 390 mg M&M? [1.5]
 - f. How many standard deviations above or below the mean are the following M&Ms?
 - i. 370 mg
 - ii. 405 mg
 - iii. 354 mg
 - iv. 312 mg [2.4 below]
 - v. 365.5 mg
 - vi. 486 mg
 - vii. 202 mg
 - g. Find the masses of M&Ms the following numbers of standard deviations above or below the mean.
 - i. 3 above
 - ii. 2 below
 - iii. 1.2 above [384 mg]
 - iv. 0.7 below
 - v. 3.8 below
 - vi. 9 above

Z scores

1. Complete the following table.

Mean μ	Standard deviation σ	x	z
360	20	400	2
360	20	310	-2.5
360	20	378	0.9
20	4	25	1.25
20	4	16.2	-0.95
20	4	4.7	3.825
50	3.2	56.1	1.906
50	3.2	44.72	-1.65
173	8.5	159.4	-1.6
173	8.5	191.5	2.176
360	20	404	2.2
360	20	325.4	-1.73
140	12	179.6	3.3
140	4	688	137
76	3.5	355.65	79.9
4.54	0.27	5.6524	4.12
0.0621	0.0112	0.0628	0.0635
360	20	380	1
360	3	354	-2
26	2	29	1.5
12.8	0.5926	11.2	-2.7
104	10	94	-1
158	20	188	1.5
83.67	4.3	92.7	2.1
438.55	27.8	429.1	-0.34

2. Find:

- a. $P(z > 1.1)$ 0.136
- b. $P(z < 2)$ 0.977
- c. $P(z < -0.6)$ 0.274
- d. $P(z > -1.8)$ 0.964
- e. $P(z > 5)$ 0.000 000 29
- f. $P(z < -5)$ 0.000 000 29
- g. $P(z > -5)$ 0.999 999 71
- h. $P(z > 0.05)$ 0.480
- i. $P(x > 42)$ if $\mu = 35$ and $\sigma = 12$ 0.280
- j. $P(x < 0.77)$ if $\mu = 0.8$ and $\sigma = 0.015$ 0.0228

3. Find a if $P(z > a) =$

- a. 0.24 0.706
- b. 0.71 -0.553
- c. 0.993 -2.46
- d. 0.082 1.392

4. Find a if $P(z < a) =$

- a. 0.41 -0.228
- b. 0.5 0
- c. 0.87 1.126

5. Find:

- a. $P(0.22 < z < 0.43)$ 0.079
- b. $P(-0.29 < z < 0.67)$ 0.363
- c. $P(0.02 < z < 0.5)$ 0.183
- d. $P(0.9 < z < 8)$ 0.184
- e. $P(-1.44 < z < -0.25)$ 0.326
- f. $P(-0.93 < z < -0.43)$ 0.157
- g. $P(20 < x < 45)$ if $\mu = 36$ and $\sigma = 12$ 0.682
- h. $P(x < 120)$ if $\mu = 125$ and $\sigma = 8$ 0.266
- i. $P(x > 9.2)$ if $\mu = 8$ and $\sigma = 2.5$ 0.316

6. Find:

- a. c if $P(0.22 < z < c) = 0.19$ 0.762
- b. c if $P(-0.62 < z < c) = 0.7$ 1.852
- c. c if $P(c < z < 1.44) = 0.5$ -0.189
- d. c if $P(c < z < -0.75) = 0.06$ -0.968
- e. c if $P(38 < x < c) = 0.01$ and $\mu = 30$ and $\sigma = 9$ 38.34
- f. c if $P(c < x < 2.2) = 0.61$ and $\mu = 1.8$ and $\sigma = 0.2$ 1.732

7. Answer these:

- a. What percentage of normally distributed scores are within 0.5 standard deviations of the mean?
- b. What percentage of normally distributed scores are within 1.2 standard deviations of the mean?
- c. What percentage of normally distributed scores are more than 2.1 standard deviations from the mean?
- d. What percentage of normally distributed scores are between 0.8 standard deviations below the mean and 0.6 standard deviations above the mean?
- e. The tallest 0.2% of the population are more than how many standard deviations above the mean?
- f. The lowest quarter of normally distributed scores are more than how many standard deviations below the mean?

a. 38.3% b. 77.0% c. 17.9% d. 51.4% e. 2.88 f. 0.674

Normal Distribution Revision Questions

- Q27. IQs of Australians are normally distributed with a mean of 110 and a standard deviation of 15. The government wants to offer an assistance program for the 10% of the population with the lowest IQs.
- What IQ should be the cut-off for receiving assistance?
 - What should be the cut-off if only 3% were to get the assistance?
- Q28. The masses of the 1729 sheep slaughtered at the Williams Brothers Meat Works in the past month were roughly normally distributed with a mean of 28.4 kg and a standard deviation of 2.1 kg. The management decide to return the smallest 5% of the sheep to the farmers rather than slaughter them.
- What should be the minimum mass for a sheep to be slaughtered?
 - What should the minimum mass be if a quarter of the sheep are to be returned?
The meat works decide to keep the heaviest 5% of the sheep and re-sell them for breeding.
 - What should be the minimum mass for re-selling?
(Hint: 95% are below the mass for re-selling.)
 - What would it be if they decided to re-sell just the heaviest 0.4%?
- Q29. The heights of Australian men are normally distributed with a mean of 176 cm and a standard deviation of 11 cm. An underpass on walkway is to be designed such that 98% of men don't have to duck to get under it.
- How high should it be?
 - How high would it need to be if they were happy for 20% of men to have to duck?
- Q30. The contents of 750 mL Fizzo's Lemonade bottles are found to be normally distributed with a mean of 750.8 mL and a standard deviation of 1.6 mL
- What percentage of the bottles contain less than the advertised contents?
 - What percentage are more than 1 mL under the advertised contents?
 - If Fizzo's could increase the mean contents without affecting the standard deviation, by what amount should they increase the mean if they want only 2% to be below the advertised contents?
- Q31. The birth masses of warthogs are normally distributed with a mean of 0.85 kg. If 68% are between 0.8 and 0.9 kg, what is the standard deviation?
- Q32. The lives of light globes were found to be normally distributed. It was also found that 44% last longer than 1000 hours and 9% last longer than 1200 hours. Find the mean and standard deviation of their lives.
- What fraction will last less than 582.09 h of use?
 - What fraction will last within 50 hours of the mean life?
In a batch of 500 bulbs, how many will last more than 1200 h?
- The mean height for 12-year-olds is 140 cm and the standard deviation is 12 cm. What is the probability that a randomly chosen 12-year-old will be between 130 and 145 cm?
 - The mean height for 14-year-olds is 155 cm and the standard deviation is 10 cm. What percentage of 14-year-olds are between 150 and 165 cm?
 - The mean height for 6-year-olds is 105 cm and the standard deviation is 9 cm. What percentage of 6-year-olds are taller than 115 cm?
 - The mean height for 12-year-olds is 140 cm and the standard deviation is 12 cm. What height would a 12-year-old have to be to be in the tallest 20% of the population?

5. The mean height for adults is 178 cm and the standard deviation is 11 cm.
If the shortest 3% of adults are dwarves, how tall does an adult have to be not to be a dwarf?
6. Annual rainfall in Waterford is normally distributed with a mean of 943 mm and a standard deviation of 205 mm. How many years in a century would the rainfall be likely to exceed 1200 mm?
7. If we say a z -score for a reading is -1.2 , what does this mean?
8. Write the formula for z in terms of x , μ and σ .
9. Write the formula for x in terms of z , μ and σ .
10. Find z if $x = 14$, $\mu = 10$ and $\sigma = 2.4$.
11. Find x if $z = -1.2$, $\mu = 85$ and $\sigma = 7.6$.
12. Find μ if $z = -0.75$, $x = 140$ and $\sigma = 12$.
13. Find σ if $z = 2$, $x = 95$ and $\mu = 88$.
14. What percentage of the population will have an IQ more than 1.2 standard deviations above the mean? What assumption do you need to make?
15. What is the probability that the value of a randomly distributed variable will be below 0.4 standard deviations above the mean?
16. What percentage of a normally distributed population will have a z -score above -0.6 ?
17. Above what z -score would you find 22% of a normally distributed population?
18. Below what z -score would you find 68% of a normally distributed population?
19. The mean mass of 60 g Mars Bars are normally distributed with mean 60.2 g. If 18% are below 60 g, what is the standard deviation?
20. The masses of Angry Mack's Whoppers are normally distributed with standard deviation 32 g. If 20% weigh more than 550 g, what is the mean mass? Comment on the reasonableness of your answer.
21. The lengths of a certain type of bolt are normally distributed. If 10% are shorter than 147 mm and 20% are longer than 154 mm, what are the mean length and the standard deviation?

Harder Problems Involving Binomial and Normal Probabilities

1. A large number of families with 6 children were surveyed.
 - (a) Tabulate the probability distribution of the random variable 'number of girls in a 6-child family'? [Assume that the probability of a child being a girl is 0.5.]
 - (b) What would be the mean number of girls and what would be the standard deviation?
 - (c) If two 6-child families were picked at random, what is the probability that they would have the same number of girls?
2. The lengths of adult millers carp are normally distributed with mean 14.5 cm and standard deviation 2 cm.
 - (a) How many would you need to catch to have a 90% chance of having at least one over 16 cm?
 - (b) What assumptions would you need to make?
- Q4. 528 people apply for roles in a movie. For the role of Titan, they will only take people taller than 180 cm; for the role of Zarg, they will only take people between 150 and 180 cm tall. The heights of the applicants are normally distributed with mean 170 cm and standard deviation 10 cm. Which role will have the most applicants and by how many?