

Glenmore State High School Mathematics Department	
Design-a-Die Assignment	
Name:	Teacher:
Year Level:	KAPS :
Unit: Probability	C&J*:
Date:	MAPS**:
UNLESS OTHERWISE INSTRUCTED, ANSWER ON LINED A4 PAPER.	DUE DATE:

The Roll-Off Game

You have a normal die. I have a normal die. We play a game of Roll- Off. We each throw our die. If your number is higher, you win a point. If my number is higher, I win a point. If we tie, no one wins a point. We each throw our die 36 times, and we record the outcome in a frequency table (see the diagram for an example).

Player	Tally	Freq
Student		
Teacher		
Tie		
TOTAL		36

Activity I – Analyzing the Roll-Off Game

1. Complete an Outcomes Table to list all of the possible outcomes in this game.
2. Ana and Bardon are playing the Roll-Off Game. They both throw their die.
 - a. Find $P(\text{Ana wins a point})$
 - b. Find $P(\text{Bardon wins a point})$
 - c. Find $P(\text{tie})$ (3, 1, 1, 1)
- 3*. Explain why this game is fair.

Activity II – Design a Better Die

A normal die has the numbers 1, 2, 3, 4, 5 and 6 on it. If you count the number of spots, you find there are $1 + 2 + 3 + 4 + 5 + 6 = 21$ spots altogether.

- 1**. Design a die that will, on average, beat a normal die in the Roll-Off Game. You may put any whole number - even zero - on any face of the die. BUT, the total of the 6 faces must be 21.

Use an Outcomes Table to show that your dice will, on average, beat a normal die.
2.
 - a. Find $P(\text{your die wins})$
 - b. Find $P(\text{normal die wins})$
 - c. Find $P(\text{tie})$ (1, 1, 1)
- 3*. If you play with your special die against a normal die, is the game fair? Explain.

Activity III – Beat the Teacher’s Die

Your teacher has designed his special die. He will give you the numbers on his special die. In this activity, you are to design a die that will, on average, beat the teacher's die.

You can put any whole number on each face - even zero. BUT - the sum of the numbers has to be 21.

1. Here are the teacher's numbers: _____
- 2**. Design a die that will beat your teacher's die. Use an Outcomes Table to show that your die will, on average, win more often than the teacher's die.
- 3*. Explain why your die is better than the teacher's die.
4. If you and the teacher each roll your die, what is the probability that
 - a. you will win?
 - b. the teacher will win?
 - c. there will be a tie? (1, 1, 1)

ACTIVITY IV – The Homework Gamble

Mr Boggs has four dice. The numbers on each die are shown below:

Die A:	<u>2</u>	<u>3</u>	<u>3</u>	<u>9</u>	<u>10</u>	<u>11</u>
Die B:	<u>5</u>	<u>5</u>	<u>6</u>	<u>6</u>	<u>6</u>	<u>6</u>
Die C:	<u>4</u>	<u>4</u>	4	<u>4</u>	<u>12</u>	<u>12</u>
Die D:	<u>0</u>	<u>1</u>	<u>7</u>	<u>8</u>	<u>8</u>	<u>8</u>

At the end of each lesson, he chooses a die. The students then choose one of the remaining die. The two dice are rolled. If the students' number is higher, they have no homework. If the teacher's number is higher, the homework is doubled.

1. Mr Boggs chooses Die A. Assuming they want no homework, use Outcomes Tables to show that the class should choose Die C. (3)
2. Mr Boggs chooses Die B. Assuming they want no homework, use Outcomes Tables to show that the class should choose Die D. (3)
- 3.** If Mr Boggs chooses Die C, which die should the class choose? Justify your answer.
- 4.** If Mr Boggs chooses Die D, which die should the class choose? Justify your answer.

BONUS QUESTION: +3 Points

Mr Boggs has decided to be generous so he has offered to change the rules of The Homework Gamble. He will let the students choose their die first, and then he chooses his die.

Should the students accept the new rules? Justify your answer.

Note: All explanations and justifications should be comprehensive and clearly written. Often a good explanation or justification includes a table, graph or diagram.

Outcomes Table

Player: _____

# on die							

Player: |

Number of times I win: _____
Number of times opponent wins: _____
Number of ties: _____

Outcomes Table

Player: _____

# on die							

Player: |

Number of times I win: _____
Number of times opponent wins: _____
Number of ties: _____

Outcomes Table

Player: _____

# on die							

Player: |

Number of times I win: _____
Number of times opponent wins: _____
Number of ties: _____

Outcomes Table

Player: _____

# on die							

Player: |

Number of times I win: _____
Number of times opponent wins: _____
Number of ties: _____

Design-a-Die - Teacher Notes

Activity I – Normal Dice

Group Size: 4 students (call them A, B, C and D)

Materials: 1 die per student, Design-a-die worksheet

Each group will hold a tournament. In round I, A and B will have a roll-off, as will C and D.

A roll-off consists of each student rolling their die at the same time. The person with the larger number wins a point. If it is a tie, neither player receives a point. The students keep track of the number of points won by each student. Roll a total of 36 times each.

The person with the most points wins. If it is a tie, roll again until a player wins.

In round II, the winners play against each other, as do the losers.

Complete Activity I on the accompanying worksheet.

Activity Two – Design-a-Die

Group Size: 4 students (call them A, B, C and D)

Materials: 1 die per student. Round sticky labels that fit onto each die face.

We will have a new tournament, with one difference - first the students can design their own die! The die may have any number on each face (from 0 upwards) but the sum of the numbers must be no more than 21. For example, A could choose to put the numbers 2, 3, 3, 3, 5, 5 on her die, since $2 + 3 + 3 + 3 + 5 + 5$ adds to 21.

Each student should write their numbers onto small sticky labels and attach these to their die.

The tournament has the same rules as before.

Complete Activity II on the worksheet.

Activity Three – Improving your die

Group Size: 4 students (call them A, B, C and D)

Materials: 1 die per student. Round sticky labels that fit onto each die face.

Design a new die that you think is the best die possible.

Complete Activity III.

Notes on Design-a-Die

Further Teaching Ideas

Some Initial Conclusions

Some dice designs will beat other dice designs in the long run, even if they each have 21 dots.

The hierarchy of dice is not transitive. That is, if die A beats die B and die B beats die C, then die A does not necessarily beat die C.

Some dice designs seem to beat many other dice designs.

Some Probability Question Frames for Students to Ask or Answer

What is the probability that a die with sides _____ will beat a die with sides _____ on a single roll?

What is the probability of rolling a sum of _____ when rolling two standard dice?

Design a pair of dice with 21 dots each that maximizes the probability of getting a sum of _____. What is that probability?

Design a die with 21 dots that will beat a die with sides _____ What is the probability that it will win on a single roll?

Some Larger Questions for Investigation

Can a die design be found that beats any given die design? How many different die designs are there?

Is there a best die design? (Best die being defined as beating more other die designs than any other.)

Is there a die design that ties all other designs?

If die A loses to die B, what is the relationship of A' to B' ? (A' , the compliment of A, has digits on its faces that are $7 - n_i$, where n_i are the digits on die A.)

How are the answers to the questions above changed if 0 is not allowed?

My Notes

This activity takes about 3-4 hours of class time.

If I can "peek" at my opponent's die first, can I build a die that will beat her?

How likely is a transitive and non-transitive relation?

Design a Winning Die

Activity I

Get into groups of size 4.

Round 1

Each person plays against an opponent, as follows:

You both roll your die at the same time. The person with the larger number gets a point. If you have the same number, record this as a tie. Keep track of how many points each of you has, as well as how many ties there are. Roll a total of 36 times.

If there is a tie at the end, have a roll-off.

Round 2

Winners play each other. Losers play each other.

To determine if this was a "fair" competition, fill in the Outcomes Table showing all possible matchups between the winning dice and your die.

Answer the questions on the worksheet.

Design a Winning Die

Activity II – Design a Winning Die

We will be having a new tournament with the same rules as before. This time, however, you make your own die. Your die may have any whole number from 0 upwards on each face. BUT the sum of the six numbers on the faces must be 21.

Round 1 My digits _____

Opponent's digits _____

Results: My wins _____

Opponent's wins _____

Ties _____

Round 2 My digits _____

Opponent's digits _____

Results: My wins _____

Opponent's wins _____

Ties _____

Design a Winning Die

Who Had the Best Die?

To determine if this was a "fair" competition, fill in the Outcomes Tables showing how your die compares to that of your opponents.

Answer the questions on the worksheet.

Activity III – Design a Better Die

Can you build a die that is better than anyone else's?

Build your die. Test it against others by completing the Outcomes Tables.

Answer the questions on the worksheet.

Activity IV – Beat the Teacher!

Can you build a die that will beat the teacher's die?

Design your die. Test it against the Teacher's Die completing the Outcomes Tables.

Answer the questions on the worksheet.

Complete the Summary at the end. Answer using full sentences.