The Birthdays Problem

Suppose we assume that birthdays are equally spread throughout the year (in other words, the probability that a randomly chosen person's birthday falls on any given date is 1 in 365 – for the purposes of this problem we'll ignore 29th February).

1. If we have a group of 20 people, what is the probability that there are at least two people who have the same birthday?
2. How big does a group need to be so that the probability of at least two people having the same birthday is more than 0.5?

To answer the first question, we need to consider the *complementary* situation. This means that:
P(at least two people have the same birthday) = 1 – P(everyone has a different birthday)

P(everyone has a different birthday) can be found as follows:
The first person's birthday can be any of 365 out of 365 days.
For the second person's birthday to be different, his/her birthday can be any of 364 out of 365 days, because one day had already been "taken" by the first person.
For the third person's birthday to be different, his/her birthday can be any of 363 out of 365 days, because two days have already been "taken" by the first and second people.
This process continues down to the 20th person.

Hence:

P(at least two people have the same birthday) $= 1 - \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \frac{362}{365} \times \ldots \times \frac{346}{365}$

≈ 0.411

To answer the second question, we continue calculating as before, adding more terms. It turns out that in a group of 23 people, the probability of a shared birthday is just over 0.5.