Year 8 AMSCO Mathematics - Probability Unit

Probability concept map. Rel Freq worksheet. HW: Finish Relative Frequency worksheet.	Probability simulations using a TI-83 Plus graphics calculator.	Law of Large Numbers program.	Probability concepts, terminology, classical definition of probability. HW: from text.

Learning Experiences

Probability Scale
Draw a probability scale (see page 26) and decide where to place words/phrases such as impossible, almost certain, even-stevens, possible, probable, a snowball's chance in hell, etc.

Experimental vs Theoretical Probability
Discuss the probability of tossing a head with a coin. Discuss probability of a thumb tack landing point up when it is tossed on a desk. In the first instance, we can say P(head) = .5, without having to perform any experiments. In the second instance, we *can't* say P(point up) = .5. Why not; what is the difference?. Have kids work in pairs tossing drawing pins (5 or 10 at a time is best) and recording results. ('Point up' actually occurs about 2/3 of the time.) Discuss the result. Distinguish between experimental probability (aka relative frequency) and theoretical probability.

Odd Shaped Spinner
Show OHT of spinner with equal central angles (so the probability of each outcome is the same), but with different areas associated with each angle (see attached sheet). For each spinner, discuss the odds of spinning a 1. Discuss.

Tossing Two Coins
In this experiment, the students toss pairs of coins repeatedly. Beforehand, have students estimate the probability that each of these will occur - 2 heads, a head and a tail, 2 tails. In pairs, have the students toss two coins 100 times each and record the results. Collate results for the class. Discuss the outcome. Show why the theoretical probabilities are 1/4, 1/2, 1/4 respectively.

Tossing Two Dice
Tossing two dice of different colours - list possible outcomes in grid form. Discuss: is 3,4 different to 4,3? Is 6,6 different to 6,6? What are the theoretical probabilities of a total of 7, total of 6, total greater than 6, etc?

Dice Difference
Introduce Dice Difference. In pairs, students play, say, 20 games. Record results. Collate results from whole class. Discuss theoretical probabilities by first drawing a grid. How well do the experiments match the theory? Change the rules to make the game more fair.

Prisoner
Introduce Prisoner. Students play a number of rounds, trying to improve their placement of of the prisoners. Discuss which strategy appears to be best. Now analyse the game mathematically. Decide on what long term strategy is best.

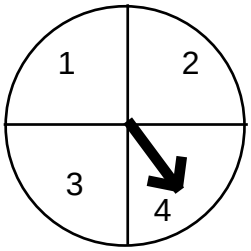
'Unders and Overs'
Introduce Unders and Overs. Play as a whole class for about 20 minutes or so. Collate winnings and losses. Discuss why the banker has come out the real winner. Analyse the game mathematically, by first drawing a grid.

Extension - The 'Coin Tossing' Carnival Game
(this is a hard question for year 9s. only challenge the best students with it)
Discuss the game (see diagram below). The diameter of a 20c piece is 24 mm and the side length of the square is 40 mm. Assuming that all coins land on the table, and in a random fashion, what is the probability that a coin will land entirely inside a square? (Hint: look at where the centre of the coin may land.) The students could solve this by experiment, by constructing a large grid, and tossing a 20c coin a large number of times and recording the results.

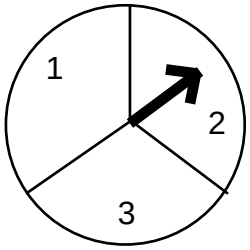
Betting Against the TAB
Discuss horse racing, and how the TAB decides on the payouts. Do the horse racing activity. Discuss the implications.

A Probability Problem

Below are two spinners. Player A has the spinner on the left. Player B has the spinner on the right. Each player spins their spinner. The numbers are then added. If the total is odd, then player A wins \$1. If the total is even, then player B wins \$1.



A



B

a. Complete the grid below, which gives the total of the two spins

		Spinner B		
		1	**2**	**3**
Spinner A	**1**	**2**	**3**	
	2			
	3			
	4			

b. Is this game fair? Fully explain your reasoning.

c. The players decide to change to rules. Now the winning player wins the total of the two dice. For example, if A spins a 2 and B spins a 3, then player B wins \$5.

Show that this game is not fair.

Unders and Overs

If two dice are tossed, the total of the two dice will be between 2 and 12. The following gambling game is proposed:

> You can bet $10 on whether the total will be Under 7, Over 7, or Equal to 7, when the two dice are next tossed.
> The payout is as follows. If you bet on Under 7, and the total *is* under seven then the payout is even money, ie you win $10.
> If you bet on Over 7, and the total *is* over 7, then again you win $10.
> If you bet on 7, and the total is *exactly* 7, then the payout is 4 to 1, ie you win $40.
> In all other cases, you lose.

Analyse this game. Which bet is the best bet to make? On average, how much would you expect to win or lose if you play the game over and over?

Explain your answer as clearly as you can.

Glenmore State High School
Mathematics Department

Name:	**Teacher:**
Year Level: Year 9	1. Show all working.
Unit: Probability	2. Answer on lined A4 paper.
Date: February 1996	3. Calculators ARE allowed.

1 2 3 4 5	**Working Mathematically**	**Rating**

1. I toss a normal 6-sided die. Find the probability that I throw a:

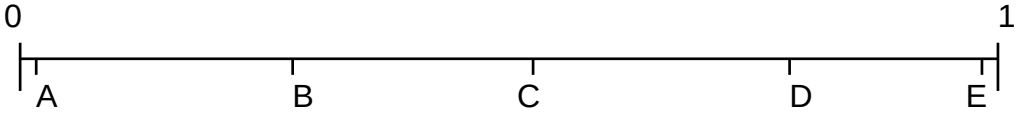
   a. four _____

   b. even number _____

   c. a number less than five _____

   (H=3/3,S=2/3)

2. Given is a probability number line. For each statement below, write the letter (A to E) that best matches the statement.



   a. It will probably rain tomorrow. _____

   b. Its about as likely as Mr Twiner becoming a jockey and winning the Melbourne Cup. _____

   c. The chance that the next baby born in Rockhampton is a girl. _____

   (H=3/3,S=2/3)

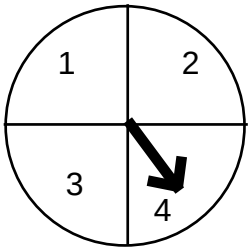
3. In a raffle with 100 tickets, here are the prizes:
   one prize of $100
   two prizes of $50
   ten prizes of $10

a) If I buy a single ticket, what is the probability that I will win the $100 prize? _____

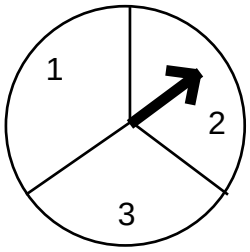
b) If I buy a single ticket, what is the probability that I will win any of the prizes? _____

(H=2/2,S=1/2)

4. Below are two spinners. Player A has the spinner on the left. Player B has the spinner on the right. Each player spins their spinner. The numbers are then added.



A



B

a. Complete the grid below, which gives the *total* of the two spins.

Spinner B

Spinner A	**1**	**2**	**3**
1			
2			
3			
4			

Find:

b. P(total is greater than 4). _____

c. P(total is not 4 or 5). _____ (H=3/3,S=2/3)

5. In the game above, if the total is odd, then player A wins $1. If the total is even, then player B wins $1. Show why this game is fair. **Fully explain your reasoning.** (H=full,S=partial explanation)

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Working Mathematically

The players decide to change the rules. Now the winning player wins the total of the two numbers. For example, if A spins a 2 and B spins a 3, then player A wins $5.

Show that this game is not fair. **Fully explain your reasoning**. (H=full,S=partial explanation)

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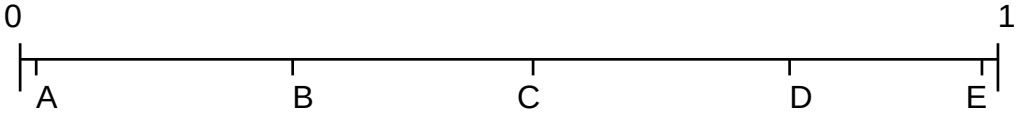
Year 9 Probability Exam - Revision Sheet

1. I toss an eight-sided die. Find these probabilities:
   a. P(4) _____
   b. P(odd number) _____

2. Given is a probability number line. For each statement below, write the letter (A to E) that best matches the statement.



____ It is highly unlikely that Mr Harrison will ever become Mr. Universe, no matter how hard he trains.

____ Mr Boggs checks homework about once a week. What is the chance that he will check it tomorrow?

3. This table shows how year 11 Maths A students get home after school.

Method	Frequency
Walk	11
Cycle	19
Bus	15
Car	10

a. What is the probability that a randomly selected student catches a bus? _____
b. If I surveyed 30 students at random, how many would I expect to cycle? _____

4. I toss two dice. I am interested in the total of the two dice.
a) Complete the grid for this probability experiment.

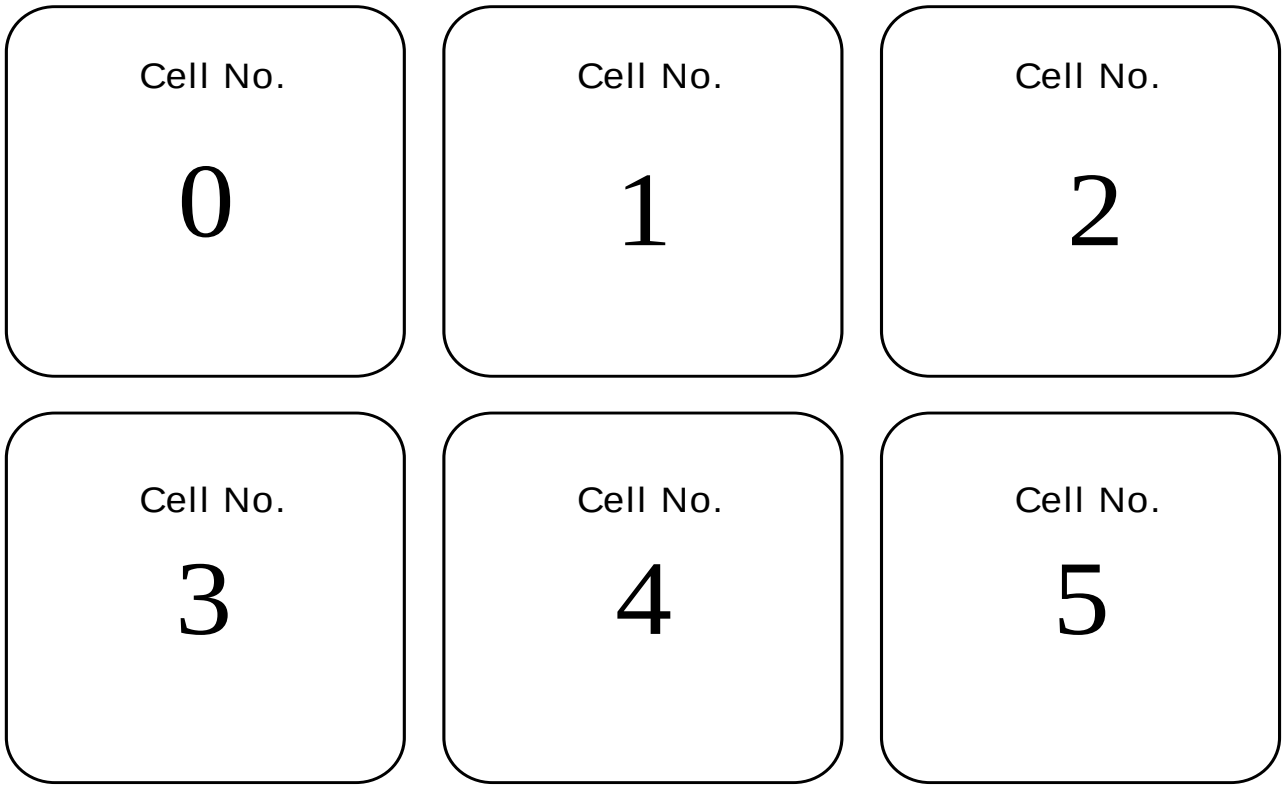
		Red die					
		1	2	3	4	5	6
Green Die	1						
	2						
	3						
	4						
	5						
	6						

Use your grid to answer the following questions.
b) What is the probability that the sum is 6? _____
c) What is the probability that the sum is greater than 6? _____
d) What is the probability that sum is either 2 or 12? _____

5. A roulette wheel has 18 red numbers, 18 black numbers and 2 green numbers. Find the probability that the next number that comes up is:
a. red _____
b. green _____
c. red or black _____
d. not black. _____

Dice Differences - Prisoners

The game and challenge ! ***How should you place your prisoners in the cells so that you can release them as quickly as possible ?***



Equipment Needed (for each player)

- a playing board of 6 cells, 6 counters (prisoners),2 dice (per pair of players)

Rules

- Each player can place their prisoners into any cells on their own game board. You can place one in each cell, or two in some cells and none in others, or even all six in one cell.
- Take turns to roll the two dice. Calculate the difference between the two numbers. You can release **one** prisoner **(only one !)** from the cell with that number. (eg. If the difference is 2, you can release **one** prisoner from cell 2.
- The winner is the first to release all their prisoners.
- Keep a record of where you place your prisoners for each game, then **tick the ones that were winners**, as shown in the table below.

	Cell						Winners
	0	1	2	3	4	5	
Prisoners	1	1	1	1	1	1	
	2	2	1	1	0	0	√

Greedy Pig

Objectives: Students get a feel for the variation that occurs in random events
Students have to make and justify decisions, based on probability
Students learn how to construct stem-and-leaf plots

Materials

1 die

Preparation

All students make a table as follows:

Round	Points
1	
2	
3	
4	
5	
TOTAL	

The Game

The match consists of 5 rounds. Each round consists of a number of games. Before each game, each student decides whether to stop (and retain the points they have earned in that round) or continue to play (and either win more points, or lose all points gained in that round, depending on the outcome).

All students get the first two throws for free. A die is tossed twice. The points are added together.

All students now stand up.

Each student now has two options.

They can quit, in which case their score is the total of the two dice. They sit down, and write this total in the Points column for round 1.

They can continue to play, and thus remain standing. The die is tossed. If the number is 1, 3, 4, 5 or 6, the student adds this number to their total for that round. But if the number is 2, the students that are still standing lose all points for that round, and record a score of 0 for that round.

The object of the game is to determine a strategy that in the long term will maximise the total points.

For each game, the student has the same option. They can decide to not risk their points, sit down and record the points they have earned in the Points column for round 1. Or they can continue to stand, the die is tossed and points are awarded (or lost) as before.

This continues until all students have sat down, or a 2 is rolled on the die. Then that round is over.

This process is continued for five rounds. The student then adds the points to get a TOTAL.

Record the Data in a Stem-And-Leaf Plot

While the students are adding their points, the teacher draws the stem of a stem-and-leaf plot on the board. Students are asked to come to the board and record their total points. The teacher explains the stem and leaf plot to the first few students, and then steps back and lets these students explain it to the others.

In a few minutes, the data is on the board, and the students have learned how to construct a stem-and-leaf plot!

Analyse the Outcome

Ask students what strategies they used to decide when to stop. Record these on the board as students speak. Discussion of the various strategies arises naturally. Some of the strategies are:

always quit after the first two throws (conservative strategy)
quit after n throws (eg I expect a 2 after six throws)
quit after n points
quit when n% of the class has sat down (don't stand out in the crowd strategy)
never sit down, hoping for an enormous score (stand out from the crowd strategy)

Compare the strategies of the 'winners'with those of the losers.

Play Again

Tell the students that they will play again, but this time they should choose a strategy first, and stick to it. Repeat the game.

Record the Data in a Back-to-Back Stem-And-Leaf Plot

Ask the students to record their result again. Show the first few students how to record their result on the left of the stem. The students have now learned how to construct a back-to-back stem-and-leaf plot!

Analyse the Outcome

In the second round, there is noticable improvement in the overall results of the class. Discuss how the back-to-back S&L plot displays this. With a senior class, you could start to discuss a theoretical approach to this problem.

Homework

Students toss a die until a 2 comes up. Record the number of throws and the total points. Repeat 20 times. Put each set of results into a frequency table.

Dice Difference

In this game, you will play against a partner. One player is the Low player, and the other is High. Each pair of players needs two dice.

You will throw two dice, and *subtract* the smaller number from the larger. The answer is called the 'Dice Difference'. The smallest difference is 0, if the two numbers are the same. The largest difference is 5, if the numbers are 1 and 6.

If the Dice Difference is 0, 1 or 2, the Low player wins. If the Dice Difference is 3, 4 or 5, the High player wins. For example, if the two dice are 5 and 2, then the Dice Difference is 3. The High player wins one point.

Play 25 rounds against your partner. Record who wins each round, and who wins overall. Does the game seem to be fair to both players?

Investigation
A good game should be fair to both players. Explain how you would determine if the game Dice Difference is fair to both players.

If the game isn't fair, explain how you could change the rules and make it fair.

Unders and Overs

White

Brown	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

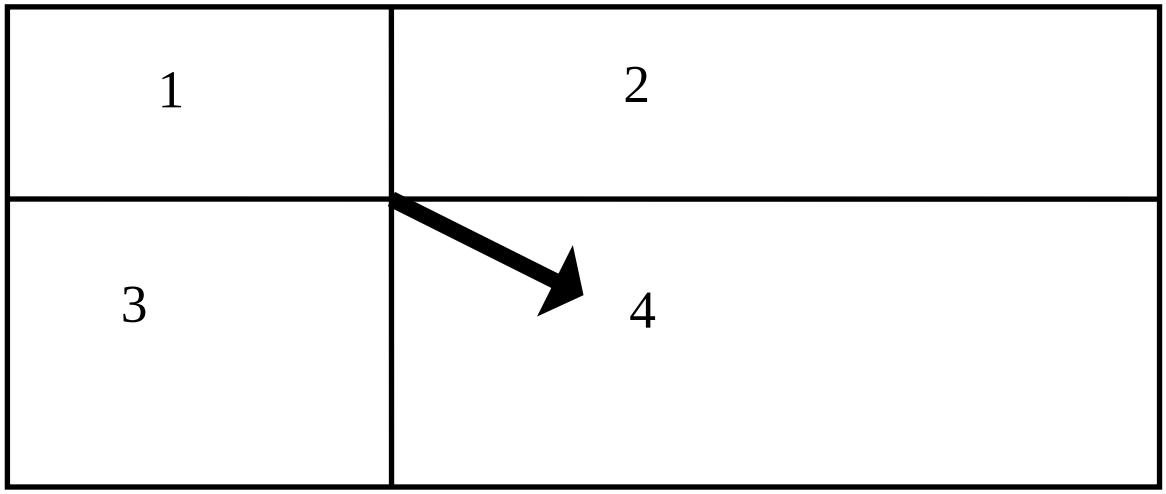
Dice Difference

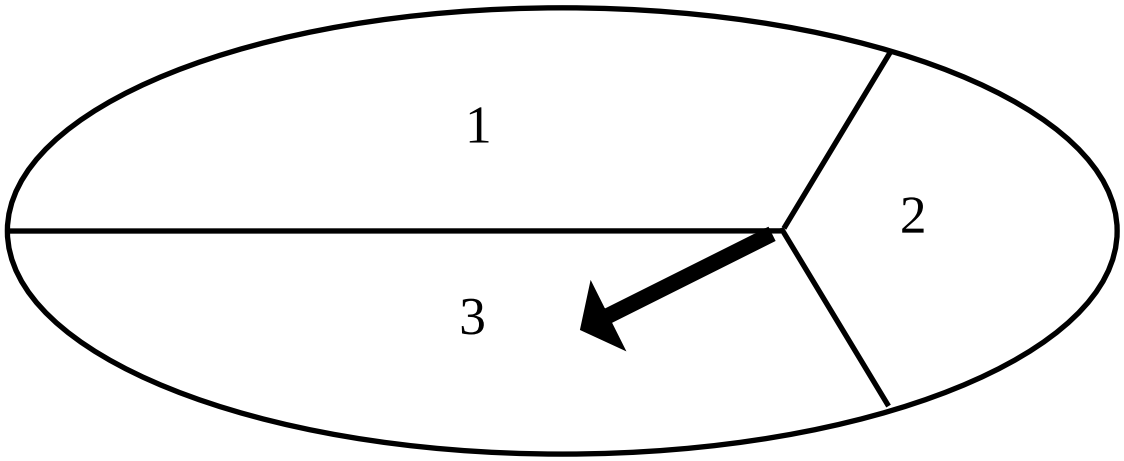
White

Brown	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Odd Shaped Spinners

What is the probability of spinning a 1?





Toss 20c - Win $1