

Real and Complex Numbers - Diagnostic Test

Q1. Put ticks in the appropriate boxes. Leave the others blank.

	-6	0.3877	$\sqrt{5}$	2π	$\sqrt{-3}$	$\sqrt{16}$	∞
Natural number							
Whole number							
Integer							
Rational number							
Irrational number							
Surd							
Real number							
Imaginary number							
Complex number							

Q2. Prove that $\sqrt{11}$ is irrational.

Q3. Simplify

(a) $\sqrt{80}$

(b) $\frac{2 + \sqrt{3}}{4 - \sqrt{3}}$

Q4. Simplify

(a) i^{51}

(b) $(3 + 2i) - (2 - i)$

(c) $(3 + i)^2$

Q5. Prove that $\overline{\overline{wz}} = \overline{wz}$ where $w, z \in \mathbb{C}$

Q6. Write $(2 - 2i)^8 \times (1 + i)$ in trigonometric form.

Q7. Solve $\frac{z+1}{z-i} = 2$

Q8. Write \sqrt{i} in Cartesian form

Solutions

Q1. Put ticks in the appropriate boxes. Leave the others blank.

	-6	0.3877	$\sqrt{5}$	2π	$\sqrt{-3}$	$\sqrt{16}$	∞
Natural number						✓	
Whole number						✓	
Integer	✓					✓	
Rational number	✓	✓				✓	
Irrational number			✓	✓			
Surd			✓				
Real number	✓	✓	✓	✓		✓	
Imaginary number					✓		
Complex number	✓	✓	✓	✓	✓	✓	

Q2. Prove that $\sqrt{11}$ is irrational.

Assume $\sqrt{11}$ is rational, i.e. it can be written as $\frac{a}{b}$ where a and b are integers with no common factors

$$\frac{a}{b} = \sqrt{11}$$

$$a = \sqrt{11} \cdot b$$

$$a^2 = 11b^2$$

$\therefore a^2$ has a factor of 11, and so a has a factor of 11

So we can write a as $11k$ where k is an integer

$$a^2 = (11k)^2$$

$$= 11^2 k^2$$

$$a^2 = 11b^2$$

$$\therefore 11b^2 = 11^2 k^2$$

$$b^2 = 11k^2$$

$\therefore b$ has a factor of 11

This contradicts the original assumption and shows that $\sqrt{11}$ is not rational.

Q3. Simplify

(a) $\sqrt{80}$

$$\begin{aligned} &= \sqrt{16} \sqrt{5} \\ &= 4\sqrt{5} \end{aligned}$$

(b) $\frac{2+\sqrt{3}}{4-\sqrt{3}}$

$$\begin{aligned} &= \frac{2+\sqrt{3}}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}} \\ &= \frac{8+4\sqrt{3}+2\sqrt{3}+3}{13} \\ &= \frac{11+6\sqrt{3}}{13} \\ &= \frac{11}{13} + \frac{6\sqrt{3}}{13} \quad (\text{optional step}) \end{aligned}$$

Q4. Simplify

(a) i^{51}

$$\begin{aligned} &= i^{48} i^3 \\ &= i^3 \\ &= -i \end{aligned}$$

(b) $(3+2i) - (2-i)$

$$= 1+3i$$

(c) $(3+i)^2$

$$\begin{aligned} &= (3+i)(3+i) \\ &= 9+6i-1 \\ &= 8+6i \end{aligned}$$

Q5. Prove that $\overline{wz} = \overline{wz}$ where $w, z \in \mathbb{C}$

Let $w = a+bi$, $z = c+di$

$\bar{w} = a-bi$, $\bar{z} = c-di$

$$\begin{aligned} \text{LHS} &= \overline{wz} = \overline{(a+bi)(c+di)} \\ &= \overline{ac-bd+(ad+bc)i} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \overline{wz} = \overline{(a+bi)(c+di)} \\ &= \overline{ac-bd+(ad+bc)i} \\ &= ac-bd-(ad+bc)i \\ &= \text{LHS} \end{aligned}$$

QED

Q6. Write $(2-2i)^8 \times (1+i)$ in trigonometric form.

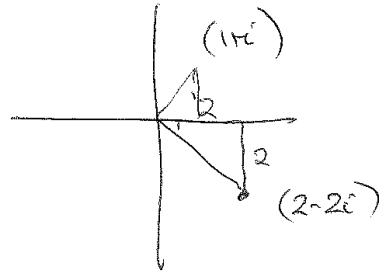
$$2-2i = 2\sqrt{2} \operatorname{cis}(-\pi/4)$$

$$1+i = \sqrt{2} \operatorname{cis} \pi/4$$

$$(2-2i)^8 = (2\sqrt{2})^8 \operatorname{cis} 0$$

$$(2-2i)^8 (1+i) = (2\sqrt{2})^8 \operatorname{cis} 0 \times \sqrt{2} \operatorname{cis} \pi/4$$

$$= 4096\sqrt{2} \operatorname{cis} \pi/4$$



Q7. Solve $\frac{z+1}{z-i} = 2$

Let z be $a+bi$

$$a+bi+1 = 2(a+bi-i)$$

$$a+1+bi = 2a+2(b-1)i$$

$$a+1 = 2a$$

$$\therefore a = 1$$

$$2(b-1) = b$$

$$\therefore 2b-2 = b$$

$$b = 2$$

$$\therefore z = a+2i$$

Q8. Write \sqrt{i} in Cartesian form

Let \sqrt{i} be $a+bi$

$$(a+bi)^2 = i$$

$$(a+bi)(a+bi) = i$$

$$a^2 - b^2 + 2abi = i$$

$$a^2 - b^2 = 0 \quad 2ab = 1$$

$$b = \frac{1}{2a}$$

$$a^2 - \frac{1}{4a^2} = 0$$

$$a^2 = \frac{1}{4a^2}$$

$$a^4 = \frac{1}{4}$$

$$a^2 = \frac{1}{2}$$

$$a = \frac{1}{\sqrt{2}}$$

$$2 \cdot \frac{1}{\sqrt{2}} b = 1$$

$$b = \frac{1}{\sqrt{2}}$$

$$\therefore \sqrt{i} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$