# Continued Fractions

There is a pretty efficient method of obtaining good rational approximations to numbers, based on continued fractions. Here's the basic idea, illustrated again with PI = 3.141592653589793+.

The first thing you do is break the number into two portions--one is the integer part, and the second is the decimal part:

```
3 + 0.14159...
```

Then you take the reciprocal of the decimal part to get

```
         1
3 + ------------
     7.06251...
```

Now repeat the above two steps:

```
          1
3 + ----------------
     7 + 0.06251...
```

```
           1
3 + -------------------
              1
     7 + -------------
          15.99658...
```

With decimal parts greater than 0.5, you can go down from above rather than up from below:

```
               1
3 + ---------------------------
                 1
     7 + ---------------------
                     1
          16 - --------------
                292.98696...
```

To turn this into a rational approximation, start from the bottom and work up. We truncate the above fraction and rewrite in one line as follows:

```
        1
3 + ----------
         1
     7 + ----
          16
```

```
       1
3 + -------
      113
     -----
      16
```

```
      16
 3 + -----
      113

  335
 -----
  113
```

Shorter truncations yield 22/7 and of course, the state of Indiana's very broad approximation, 3/1.  Good approximations are revealed by a large jump in the size of the numbers in the *following* approximation. For example, the first four fractions in the series for pi are

```
      3/1          3.000000000000000
     22/7          3.142857142857143
    355/113        3.141592920353983
 104348/33215      3.141592653921421
```

The median increase in numerator and denominator is about 4 times.  The average increase, believe it or not, is undefined--it is infinite.