Teaching Integer Operations with Understanding

Addition

Integer addition can be taught with understanding, and without a rule, by using Money and Bills.

Money is positive. So +5 represents $5 in a pocket. Bills (or debts) are negative. So –4 represents a bill for $4 in another pocket.

Examples:

> I have $5 in one pocket and $3 in the other pocket. How much altogether? Ans: +5 + (+3) = +8
>
> I have $5 in one pocket and a bill for $3 in the other pocket. How much altogether? Ans: +5 + (-3) = +2
>
> I have a bill for $5 in one pocket and $3 in the other pocket. How much altogether? Ans: -5 + (+3) = -2
>
> I have a bill for $5 in one pocket and a bill for $3 in the other pocket. How much altogether? Ans: -5 + (-3) = -8

Give students plenty of practice.

Reverse the question - give students the math, e.g. (-7) + (-2) + (-6) = -15, and have the students write the question ("I have a bill for $7 … etc).

Include problems with decimals.

Subtraction

Teach subtraction using two-color counters. After a while have the students just draw the pictures for the counters, e.g. (+) for +1 and (-) for –1 (where the parentheses represent circles). Finally have the students try to solve the problems by visualizing the counters, so they can just write the answer.

If that is conceptually too difficult for some students, develop the rule for subtracting by putting a subtraction question and the related addition questions side by side

Examples

5 - 3 = 2	←→	5 + -3 = 2
4 - 3 = 1	←→	3 + -1 = 4
2 - 5 = -3	←→	2 + -5 = -3
3 - -2 = 5	←→	3 + 2 = 5

-3 - -1 = -2 ←→ -3 + 1 = -2

-2 - -4 = 2 ←→ -2 + 4 = 2

Students will see that every subtraction question can be re-written as an addition question, by changing subtraction to addition, and the 2nd number to its opposite.

NB This illustrates the *very* powerful idea of changing a problem we can't solve into a problem we can solve. As Leibnitz said, "Every problem I solve becomes a tool for solving another problem."

Multiplying Integers

Positive x negative
Give each group a set of problems to solve, where the first number is a different positive integer for each group. The example below uses **3**. Each group shares their answers. Look for the pattern.

3 x 3 =
3 x 2 =
3 x 1 =
3 x 0 =
3 x –1 =
3 x –2 =
3 x –3 =

Discuss the answer to questions such as

-3 x 3 =
-2 x 3 =
-1 x 3 =

so students realize that order doesn't matter.

Negative x negative
Give each group a set of problems to solve, where each group has a different first negative number. In the example below, the negative number is **–3**. Each group shares their answers. Look for the pattern.

-3 x 3 =
-3 x 2 =
-3 x 1 =
-3 x 0 =
-3 x –1 =
-3 x –2 =
-3 x –3 =

Students should be able to discover that a negative times a negative gives a positive.

Dividing integers

Use the idea of number fact families, e.g.

If $3 \times 2 = 6$ then $6 \div 2 = 3$ and $6 \div 3 = 2$

So…

If $3 \times -2 = -6$ then $-6 \div -2 = 3$ and $-6 \div 3 = -2$

Have students work in groups to write a number of integer fact families. Then have the groups share. From this, the students find the "rule" for dividing integers.

Finally have students compare the rules for multiplying and dividing, and combine these rules into just one rule.