Tests of Divisibility

How can you tell whether a whole number is divisible by another number (leaving no remainder) without actually doing the division? Here are some quick ways to tell.

Divisibility by:

1 All numbers are divisible by 1.

2 If the last digit is even, the number is divisible by 2.

3 If the sum of the digits is divisible by 3, the number is also.

4 If the last two digits form a number divisible by 4, the number is also.

5 If the last digit is a 5 or a 0, the number is divisible by 5.

6 If the number is divisible by both 3 and 2, it is also divisible by 6.

8 If the last three digits form a number divisible by 8, then so is the whole number.

9 If the sum of the digits is divisible by 9, the number is also.

10 If the number ends in 0, it is divisible by 10.

12 If the number is divisible by both 3 and 4, it is also divisible by 12.

15 If the number is divisible by both 3 and 5, it is also divisible by 15.

16 If the last three digits form a number divisible by 16, then so is the whole number.

18 If the number is divisible by both 2 and 9, it is also divisible by 18.

20 If the number is divisible by both 4 and 5, it is also divisible by 20.

Here are Some More Tests of Divisibility. These aren't as quick, so are not used as often. Try them out!

7 Chop off the last digit, double it, and subtract it from the rest of the number. If the answer is divisible by 7, then the number is also.

11 Alternately subtract and add the digits from left to right. If the result is divisible by 11, the number is also.

13 Chop off the last digit, double it, double it again, and add that to the rest of the number. If the answer is divisible by 13, then the number is also. OR

Chop off the last digit, multiply it by 9, and subtract that from the rest of the number. If the answer is divisible by 13, then the number is also.

14 If the number is divisible by both 2 and 7, it is also divisible by 14.

17 Chop off the last digit, multiply it by 5, and subtract it from the rest of the number. If the answer is divisible by 17, then the number is also.

19 Chop off the last digit, double it, and add it to the rest of the number. If the answer is divisible by 19, then the number is also.

Test of Divisibility for Any Number

The best divisibility tests in general are of the form k -jm. If the number N has n digits, then k is the number formed by the first n-1 digits, m is the units digit, and j is a integer.

The trick is to figure out the value of j.

A divisibility test for 17 is k-5m, hence j = 5. If k-5m is a multiple of 17, then the original number is also. Example: Consider N = 1836. k = 183, j = 5, m = 6. k - jm = 183 - 30 = 153. Repeat if necessary. k = 15, j = 5, m = 3, so k-jm = 0. Hence 1836 is divisible by 17.

Now, why is j = 5 in this test of divisibility?

First, find the smallest multiple of 17 that is 1 more or 1 less than a multiple of 10. So, start listing multiples -- 17, 37, 51 - ok, use 51.

(An easier way of saying the same thing - the multiple has to end in a 9 or a 1.)

Now we need a bit of algebra, and number theory...

N = 10k + m by the definition of k and m
-5N = -50k - 5m multiply by -5 (to give me -50k)
51k - 5N = k - 5m add 51k to both sides (to give me k)

Now if N is a multiple of 17, so is k -5m! Proof: If N is a multiple of 17 then N can be written in the form 17p. So the equation becomes:
51k - 85p = k - 5m
17(3k - 5p) = k-5m
Therefore k - 5m is a multiple of 17.

So, what is the divisibility test for 19? Use the fact that 19 x 1 = 19, which is 1 less than 20.

N = 10k+ m as before
2N = 20k + 2m multiply both sides by 2 (to get 20k)
-19k + 2N = k + 2madd -19k to both sides. (to get k)
Therefore, if N is a multiple of 19, so is k + 2m. The proof is similar.

Check: Assume N = 1653. k + 2m = 165 + 6 = 171. Repeat using 171.
k + 2m = 17 + 2 = 19.

So a divisibility test for 19 is k + 2m. I used the fact that 19 = 1 x 19 to derive this test. I could also have used the fact that 38 = 2 x 19, to get this test - 2k + 4m, in other words, double k and add 4 times the last digit. N = 1653. 2k + 4m = 330 + 12 = 342. Repeat: 2k + m = 68 + 8 = 76. 76 is 19 x 4, therefore 1653 is also. Obviously, this test is more difficult than the first one. But it still works.

Here is your homework. Find a divisibility test for a) 23 and b) 37. Good luck!