

Why 1 is NOT a prime number

1 is *not* prime because most of the mathematics that involves prime numbers doesn't make any sense if 1 is considered prime.

Think, for example, about the fundamental theorem of arithmetic, which says that any number can be decomposed into a unique set of prime factors. 6 can be decomposed into a 3 and a 2, 24 into three 2s and a 3, and 30 into a 2, a 3, and a 5, and so on. In any case, the order of the prime factors can be switched around ($6=2\times 3$ or $6=3\times 2$), but the number of factors is invariant. 6 has two prime factors, no matter which way you write them.

If 1 is allowed, however, this is no longer true. 6 can have 2 factors (2×3), 3 factors ($2\times 3\times 1$), four factors ($2\times 3\times 1\times 1$), indeed any number of factors bigger than two. In fact, it's not too much of a conceptual leap to say that 6 could then have an infinite number of prime factors. Finite numbers containing an infinite number of primes makes for sticky number theory, so we exclude 1 as prime.

As another example, it seems pretty obvious that the product of two prime numbers is composite, since it has those two prime numbers as factors. If 1 is prime, would 17 be prime? Since $17=17\times 1$, 17 is the product of two primes, therefore 17 would be composite -- but we all know 17 is prime. Quite a contradiction. All this confusion is avoided by disallowing 1 in the set of prime numbers.