Whelk Bashing

Part I: Make a Conjecture

Sea gulls and crows feed on various types of mollusks by lifting them into the air and dropping them onto a rock to break open their shells. Biologists have observed that northwestern crows consistently drop a type of mollusk called a whelk from a mean height of about 5 meters. The crows appear to be selective; they pick up only large-sized whelk. They are also persistent. For instance, one crow was observed to drop a single whelk 20 times. Scientists have suggested that this behavior is an example of decision-making in optimal foraging.

Why do you think crows consistently fly to a height of about 5 meters before dropping a whelk onto the rocks below?

Think About This Situation

Consider the dropping of large whelks by northwestern crows.

1.Which flight path A or B do you think the crows use most? Why?

2.What factors do you think influence the height at which the crows choose to drop the whelk?

3.Do you think there is a minimum or maximum number of drops required to break a whelk?

4.Do you think there is a minimum or maximum height at which a whelk can be dropped to break?

5.What classroom experiment could model the dropping of whelks to collect and analyze data?

What questions would you attempt to answer in your experiment? How would the relationship between the number of drops and the height of the drops help you answer your questions?

6.Sketch a possible graph of the number of drops required to break a whelk as a function of the height of the drop. How are your answers above evident in your graph?

Part II: Conduct an Experiment

Are the crows minimizing their work in dropping a whelk? The amount of work depends upon the height of the drop and the number of times the crow has to fly to this height. To answer the question, the relationship between the height of the drop and the number of drops is needed.

Reto Zach * conducted the following experiment. He repeatedly dropped a whelk from a fixed height until the whelk broke. He recorded the height and the number of drops required. He repeated this for several different heights. The dropping of whelk can be simulated by dropping peanuts or other objects. Peanuts are a good choice because they are relatively inexpensive and fairly uniform.

If you do not wish to conduct an experiment to gather your own data, then use the sample data provided.

The Experiment: To model the dropping of whelks, get a meterstick and a cup of whole, blanched peanuts that have been removed from their shells. Start with a height of 15 cm. Repeatedly drop a peanut until it breaks into two pieces. Record the number of drops needed for the peanut to break. Repeat this experiment for at least eight peanuts at this same height. Find the mean number of drops required to break open a peanut. Repeat this experiment for heights of 20, 25, 30, 35, 40, 50 and 60 centimeters. You may want to pool your data with data obtained by other groups in the class. Record the data in a table similar to the following.

Reflect on the Experiment

Examine the patterns in the data that you have produced. Compare your findings to the conjectures that you made previously.

1.Are your conjectures confirmed or disputed? What changes in your conjectures would you make?

2.At which height should more peanuts be dropped to get more accurate data? Why?
How is this fact evident in your data?
How can you decide when enough peanuts at a given height have been dropped?

3.Do you think there a minimum number of drops required to break open a peanut?

4.Do you think there is a minimum height required to break open a peanut?

5.What type of function do you think could be used to model the data that you've collect?

Part III: Analyze the Data

The amount of work in dropping a whelk to break it open depends on the height of the drop and the number of times a whelk has to be dropped.

Work = Height * Number of Drops

W = H * N

To investigate the work solely as a function of the height, a relationship between the number of drops and the height of the drop is required. You may have observed that the data for (H, N) resembles the graph of the hyperbolic function which has a horizontal asymptote of x = 0 and a vertical asymptote of y = 0. The general equation for hyperbolic graphs is:

The goal of this activity is to find a good model for the relationship between the height of the drop H and the number of drops N.

For this investigation, you will need to specify a data set to use. The last data set you have been using may already be specified. The sample peanut data is from the Conduct an Experiment activity.

Reto Zach carried out an experiment of dropping whelks of various sizes from different heights. Since northwestern crows feed only on the largest whelk, the data from his experiment for the large whelk are provided (See Extending Task below).

Sample Peanut Data

Ht drop	15	20	25	30	35	40	50	60
No drops	17.3	9.25	7.13	5.13	4.15	3.25	2.63	2.25

Sample Whelk Data, from Reto Zach.

Ht drop	1.5	2	3	4	5	6	7	8	10	15
No drops	56	20	10.2	7.6	6	5	4.3	3.8	3.1	2.5

Graphical Analysis

Using the interactive graph, try different values of a, b and c in the equation $y = a + \frac{b}{x - c}$ to find a graph that fits the data..

Based on your work with the interactive graph, answer the following questions.

1.How do the values of a, b and c change the shape of the graph?

2.Based on the situation, what is a reasonable conjecture and possible explanation for the most likely value of a?

3.Using this value of a, what are values for b and c so that the function closely models the data?

4.For which data points, is getting the model to fit the most difficult? What explanation can you give

for this observation?

5.Explain why a function of the form:

$$y = a + \frac{b}{x - c}$$

is a reasonable conjecture for the data.

Experimenting with different values of the parameters a, b and c is not a systematic method for producing a model. Different people may settle on different values of a, b and c. A good method would be reproducible by anyone and based on robust procedures.

Linear regression is a widely accepted and reproducible. If the value of a can be assumed to be equal to 1, then it is possible to transform the question of finding this hyperbolic model into a finding a linear model.

$$N = 1 + \frac{b}{H - c}$$

$$N - 1 = \frac{b}{H - c}$$

$$(N - 1)^{-1} + \frac{H - c}{b}$$

Why does the last equation show that a linear relationship should exist between the H and (N - 1)-1 assuming that the first equation is a correct model?

When ready, click on the following button to show the transformed data and the corresponding graph. A line of best fit is automatically calculated for you and plotted with the data.

1.Find the equation of the line for the relationship between H and (N - 1)-1. Rewrite this equation to express the hyperbolic relationship between H and N.

Compare your equation using this method to the equation you found above. Be sure to write your equations using the variables H and N.

2.Why do you think it is necessary to assume a value for a to be able to use this method?

3.What are the horizontal and vertical asymptotes for your equations? Do these values make sense?

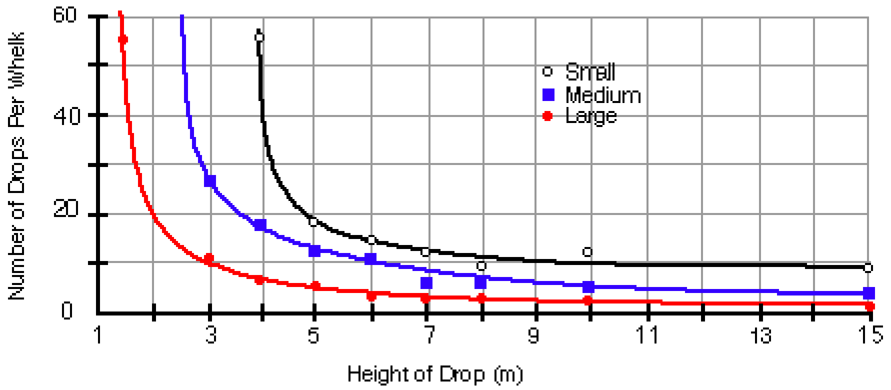
Own Your Own

Reto Zach collected data on different sizes of whelk based on weight. The graph of his results and the curves he sketched by hand are shown in the following diagram.

1.What do you observe about the vertical and horizontal asymptotes for the different sizes of whelks? Explain why the asymptotes might differ for the different sizes of whelks.

2.What are possible equations for each size of whelk?

3.Using the data for the largest whelk (See the Data Table above), find an equation using the methods of this investigation.



Part IV: Work to a Conclusion

The amount of work in dropping a whelk to break it open depends on the height of the drop and the number of times a whelk has to be dropped.

Work = Height * Number of Drops

W = H * N

If you've been working with a data set, the data should automatically be filled in below. Otherwise, enter your own data or select one of the two other data sets.

Based on the computed values of work for the individual data points, between what heights is the amount of work the smallest?

The relationship between the height of the drop and the number of drops can be used to investigate the work. Using the method of transforming the data and using linear regression, the relationship between H and N for the sample peanut data is:

$$N = 1 + \frac{59.5}{H - 13.8}$$

The equation for the amount of work for the sample peanut data is:

$$W = H \times N = H\left(1 + \frac{59.5}{H - 13.8}\right)$$

Use the function grapher or your own graphics calculator to find the height corresponding to the minimum work.

1.What is the height at which the minimum work occurs? How do values for the work near this height compare to the minimum work?

2.How does the location for the minimum work you found using the equation, compare to the value you observed from the data? Which finding do you think should be reported and why?

3.What is true about the work for large heights? Give an explanation for your observations.

4.What are the asymptotes for the work equation?

Both the expression for the number of drops in terms of the height and the work in terms of the height are rational expressions. Rational expressions are those that can be expressed as the quotient of two polynomials.

Rational expressions can be written in several different forms. Three common ways of writing a rational expression are show below for the same rational expression:

Standard Form: $\frac{x^2 - 1}{x^2 - 2x + 1}$

Factored Form: $\frac{x + 1}{x - 1}$

Proper Fraction Form: $1 + \frac{2}{x - 1}$

Each form of a rational expression provides different information and insight into the nature of the function.

Activity 2

Rewrite your expressions for N and W in each of these forms. Graph each function and determine features of the graph such as asymptotes, zeros, or the general shape.

1.What information can you determine about the function from each form?

2.Examine the proper fraction form. What does this form tell you about the amount of work for very large heights? for very small heights? Hint: What is the major contributing factor to the amount of work required in each case?

3.Write a definition for when a rational function f(x) is written in proper fraction form by completing the following lead in sentence by giving conditions on the polynomials p(x), r(x) and q(x).

If f(x) is a polynomial written in proper fraction form using the polynomials p(x), r(x) and q(x) such that $f(x) = p(x) + \frac{r(x)}{q(x)}$, then ...

Reflection Questions

Examine your original work equation and the different ways of expressing the same expression for N and W.

1.Some biologist hypothesize that the dropping whelk from a height of 5 meters by the crows is an example of optimal foraging. Using the large whelk data, would support or refute this claim? Give specific evidence.

2.Work also depends on the weight of the object. Northwestern crows drop only large whelk which are fairly comparable in weight of about 8.8 grams.

How would you alter the equation to include the factor of weight?
What happens to height at which the work is a minimum if the weight is included?
3.How was knowledge about rational functions useful in finding a mathematical model for the work involved?

Do Birds Know Calculus

Objective: Students will recognize a non-linear equation in nature. Students will get a sense of nature's natural instincts of mathematics. Students will be able to graph non-linear application and make conclusions using the graph.

Introduction: There are certain species of birds, namely gulls and some crows, that eat shellfish called "whelk." In order to eat the whelk they need to break the shells open. To do this they fly up 5 meters and drop the whelk upon the rocks. Sometimes they break open, sometimes not, but the birds continue to do this until the shell breaks. The interesting thing to note is that they always fly to 5 meters, no more, no less. Why? We will attempt to assimilate this activity using peanuts instead of whelk and try to make some conclusions.

Materials needed: Peanuts and meter sticks

Lab: Bring the peanuts up to heights of 5 cm, 10 cm, 20 cm, 30 cm, 40 cm, 50 cm, 60 cm, 70 cm, and 80 cm. Count the number of times it takes for the peanuts to split in two. Keep track of the number of drops that correspond to each height.

Graphing: Graph your heights on the x-axis and your drop # on the y axis.

Your assignment: Write up a lab report with the following:

1. A statement of the problem
2. A summary of your lab
3. Your data
4. Your graph
5. A summary answering the following questions:

   a. Is there any height at which the peanut does not seem to break?

   b. Is there a minimum number of times it needs to be dropped?

   c. Using work = height X drop number, what is the minimum amount of work? At what height does this happen?

   d. Try to develop a formula using y(drop) = ? 1/x(height)

   e. What can you conclude about the birds and the whelk?

   f. BE COMPLETE

State the problem:

What did you do?

Height	# of Drops	Work
5		
10		
20		
30		
40		
50		
60		
70		
80		

Formula:

Conclusions: