Quadratic Models

Quadratic equations arise in many real-life applications. Area, projectile motion and revenue are just some of the situations where quadratic equations are used. Below are four applications of quadratic equations. Answer all of the questions in your quad-ruled Maths pad.

1. The canonical quadratic equation is $y = x^2$.

    a. Copy and complete the following table

X	-3	-2	-1	0	1	2	3
X^2							
y							
(x,y)							

    b. Hence graph the quadratic equation $y = x^2$.

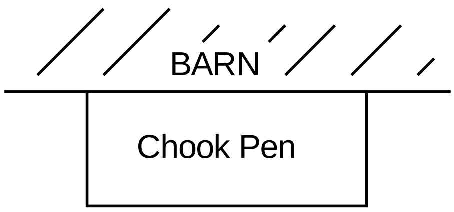
2. More complicated equations can be graphed by using a slightly more complicated table. Draw the graph of $y = x^2 - x - 6$.

    a. Complete the following table.

X	-4	-3	-2	-1	0	1	2	3
x^2								
-x								
-6								
y								
(x,y)								

    b. Draw the graph of $y = x^2 - x - 6$.

3. Farmer Twiner wishes to construct a rectangular chook pen, using his barn for one of the sides. The other three sides will be constructed out of 80 m of chicken wire (see diagram).



He can make his pen a variety of shapes - long and thin, short and fat, square or any shape in between. We want to determine which rectangular shape will give him the maximum area.

a) Let x represent the width of the chook pen. Given that Farmer Twiner has 80 metres of chicken wire, what is the length of the pen, in terms of x?

b) Write the algebraic function that gives the area of the pen, in terms of x.

c) What is the domain? (recall that the domain is the set of all possible values of x.)

d) For what values of x is the area equal to zero? Answer this by solving a quadratic equation.

e) The area of the chook pen is a function of x. Make a table that gives the area for differing values of x.

f) Draw the graph of this function.

g) From your graph, what is the maximum area of the chook pen?

4. A cricket ball is hit directly upwards at a speed of 20 m/s. The height, h, of the cricket ball above the ground after t seconds is given by the equation:

$$h = 20t - 5t^2$$

a) Make a table that shows that height of the cricket ball after t seconds.

b) Draw a graph that shows that height of the cricket ball after t seconds.

c) What is the maximum height that the cricket ball will reach?

d) Estimate from your graph the height of the ball after 2.5 seconds.

e) Calculate the height of the ball after 2.5 seconds, using the equation.

f) When is the cricket ball exactly 15 metres above the ground? Estimate the answer from your graph.

g) When is the cricket ball exactly 10 metres above the ground? Estimate the answer from your graph.

5. A large holiday complex contains 100 units. If the rent is $300 per week, all units will be rented. However for each additional $20 in rent, 5 additional units will become vacant.

a) Write an equation that relates rent, C, (cost of renting a unit for a week) and the demand, D (number of units rented).

b) Find the equation relating total weekly rental income, R, with weekly rental rates.

c) At what weekly rent per unit is the total weekly rent maximized? (You will have to choose your method of solving this mathematically)

d) What is the maximum weekly rental income?

The Tricycle Problem

A manufacturer of tricycles finds that the number of tricycles sold, *n*, is related to the price per tricycle, *p*, by the equation

$$p = -2n + 200$$

a) What price must be set if the manufacturer wishes to sell 30 tricycles?

b) How many tricycles will be sold if the price per tricycle is $80?

c) Total revenue, R(n), from sales is calculated by:

R(n) = (price per tricycle) x (number of tricycles sold)

Write a function R(n) in terms of n.

d) What is the total revenue if the price per tricycle is $80? Answer by substituting into the function R(n).

e) Graph the revenue function. You'll probably have to determine the domain first.

f) Determine from the graph:

. the maximum sales revenue;

. how many tricycles must be sold in order to maximize sales.

Model Solution with Notes

Graph $y = -2x + 200$ on your graphics calculator. Note that as the number of tricycles sold goes up, the price goes down. To sell lots of tricycles, you will need to lower the price.

a) Let n = 30
Then p = -2(30) + 200 {substitute 30 for n}
p = -60 + 200 = 140 {evaluate}
In order to sell 30 tricycles, you will have to set the price at $140.

b) Let p = 80
Then 80 = -2n + 200 {substitute 80 for p}
80 – 200 = -2n {take 200 for each side}
-120 = 2n {simplify}
n = 60 {divide by sides by -2}
If you lower the price to $80, you will increase your sales to 60 tricycles.

c) R(n) = (price per tricycle) x (number of tricycles sold)
R(n) = p x n {substitute}
R(n) = (-2n + 200) x n {substitute -2n + 200 for p}
$R(n) = -2n^2 + 200n$ {expand}

d) $R(80) = -2(80^2) + 200(80)$ {substitute}
= -2(6400) + 16,000 {simplify}
= -12,800 + 16,000
= 3,200
The revenue will be $3,200.

e) Graph it on your graphics calculator. {try Zoom Fit, or change your [Window] }

f) Choose [2nd] [Calc] [Max]. The x-coordinate is the number of tricycles sold (n = 50).
The y-coordinate is the Revenue [R(50) = 5000].
You will maximise your Revenue at $5,000 if you sell 50 tricycles.