Proving Trig Identities

**Note: Square root symbols are denoted with SQRT.

ADDITION & SUBTRACTION FORMULAS

1. sin (a + b) = (sin a) (cos b) + (cos a) (sin b)
2. sin (a - b) = (sin a) (cos b) - (cos a) (sin b)
3. cos (a + b) = (cos a)(cos b) - (sin a)(sin b)
4. cos (a - b) = (cos a)(cos b) + (sin a)(sin b)
5. tan (a + b) = [(tan a) + (tan b)]/[1 - (tan a)(tan b)]
6. tan (a - b) = [(tan a) - (tan b)]/[1 + (tan a)(tan b)]

DOUBLE ANGLE FORMULAS

7. To derive #7, use rule #1. Let a=x and b=x. This results in ==>
sin (x+x) = (sin x)(cosx) + (cos x)(sin x)
sin2x = 2 (sin x) (cos x)

8. To derive # 8, use rule #3. Let a=x and b=x. This results in ==>
cos(x+x) = (cos x)(cos x) - (sin x)(sin x)
cos2x = $\cos^2$x - $\sin^2$x

9. To derive #9, use rule #8 and the following Pythagorean identity: $\sin^2$x + $\cos^2$x = 1.
cos2x = $\cos^2$x - $\sin^2$x
cos2x = $\cos^2$x - (1 - $\cos^2$x)
cos2x = 2$\cos^2$x - 1

10. To derive #10, use rule #8 and the following Pythagorean identity: (sinx)^2 + (cosx)^2 = 1.
cos2x = $\cos^2$x - $\sin^2$x
cos2x = (1 - $\sin^2$x) - $\sin^2$x
cos2x = 1 - 2$\sin^2$x

11. To derive #11, use rule #5. Let a=x and b=x. This results in==>
tan (x+x) = [tan x + tan x] / [1 - (tan x)(tan x)]
tan2x = (2 tan x)/(1 - $\tan^2$x)

"SQUARED" FORMULAS

12. To derive #12, use rule #10 and solve for $\sin^2$x.
cos2x = 1 - 2$\sin^2$x
2$\sin^2$x = 1 - cos 2x
$\sin^2$x = (1 - cos 2x)/2

13. To derive # 13, use rule #10 and solve for $\cos^2$x.
cos 2x = 2$\cos^2$x - 1
2$\cos^2$x = 1 + cos 2x
$\cos^2$x = (1 + cos 2x)/2

14. To derive # 14, use 12 and 13.
$\tan^2$x = [(1 - cos 2x) / 2] / [(1 + cos 2x) / 2]
$\tan^2$x = [1 - cos2x]/[1 + cos2x]

HALF ANGLE FORMULAS

15. To derive # 15, use #12 and let x = y/2.
$\sin^2(y/2) = [1 - \cos 2(y/2)]/2$
$\sin (y/2) = \pm$ SQRT [(1 - cos y)/2]

16. To derive #16, use #13 and let x = y/2.
$\cos^2(y/2) = [1 + \cos 2(y/2)]/2$
$\cos (y/2) = \pm$ SQRT [(1 + cos y)/2]

17. To derive #17, use #14 and let x = y/2.
$\tan^2(y/2) = [1 - \cos 2(y/2)] / [1 + \cos 2(y/2)]$
$\tan(y/2) = \pm$ SQRT [(1 - cos y) / (1 + cos y)

18. To derive #18, multiply (under the radical) the numerator and denominator of #17 by the conjugate of the denominator.
tan(y/2) =± SQRT [[(1 - cos y)(1 - cos y)] / [(1 + cos y)(1 - cos y)]]
tan(y/2) =± SQRT [[(1 - cos $y)^2$] / [1 - $\cos^2 y$]]
tan(y/2) =± SQRT [[(1 - cos $y)^2$] / [($\sin^2 y$]]
tan(y/2) = (1 - cos y) / (sin y)

19. To derive #19, multiply (under the radical) the numerator and denominator of #17 by the conjugate of the numerator.
tan(y/2) =± SQRT [[(1 - cos y)(1 + cos y)] / [(1 + cos y)(1 + cos y)]]
tan(y/2) =± SQRT [[(1 - $\cos^2 y$]/[(1 + cos $y)^2$]]
tan(y/2) =± SQRT [[$\sin^2 y$]/[(1 + cos $y)^2$]]
tan(y/2) = (sin y)/(1 + cos y)