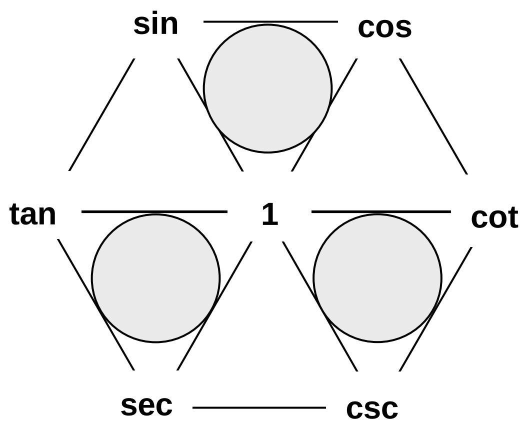
MAGIC Hexagon for Trigonometric Relations



Complementary or Co-Functions

The three functions on the left are, historically, the three principal trigonometric functions. The three on the right, all beginning with the syllable CO, are related to the ones on their left. Cosine is short for "COmplimentary sine, and reveals one of the relations in the hexagon.

Functions on the left	Functions on the right
Sin(40^0)	= Cosine (50^0)
Tan (25^0)	= Cot (65^0)
Sec (80^0)	= Cosec(10^0)

And in general:

Sin (x)	= Cos (90^0 - x);
Tan (x)	= Cot (90^0 - x)
Sec (x)	= Csc (90^0 - x)

Reciprocal Functions

In the diagram above, the ratios that are on opposite sides of the "1" are reciprocals. This is clear with example values, so :

sin(30^0) = ½	csc(30^0)= 1/sin(30^0) = 1/(1/2) = 2
cos(30^0) = sqrt(3)/2	sec(30^0) = 1/cos(30^0) =

Since the typical calculator has the functions sin, cos, and tan, to find the other three functions, we must use the reciprocal of one of the three given functions.

Pythagorean Identities

Many students find it difficult to remember the three relationships with squared values that are called the Pythagorean identities. In the geometric views of the trigonometric values as lengths on the unit circle, these three identities are the shown as sides of right triangles, hence the name. The three triangles that are shaded in with dark lines above are each one identity. First note that EACH of the three triangles is oriented so that there are two symbols on the top and one on the bottom. In each case it is true that the sum of the squares of the top two equal the square of the bottom, again assuming that all three values are defined on a common angle A.

Look at the top shaded triangle which has the Sin, Cos and 1 at it's corners. For a given angle A, this says that Sin^2(A)+ Cos^2(A)=1^2. Since 1^2=1 figures in each of the three identities, it is usually just read as "one", rather than "one squared".

Looking at the bottom left triangle, which uses Tan, 1, and Sec, we again add the squares of the two on top to get the bottom squared, giving us Tan^2(x) + 1 = Sec^2(x). It is easy to see that the bottom right triangle should yield 1 + Cot^2(x) = Csc^2(x)

Of course you learned long ago that sometimes we want to express a Pythagorean relationship as the difference between a hypotenuse and a leg, such as C^2-A^2=B^2. This happens in Trig too, but is a simple adjustment. If you start at the bottom of a triangle and go up either leg, you subtract. This gives relations like 1-Cos^2(x)= Sin^2(x), or Sec^2(x)-Tan^2(x)=1. With practice you get very quick at these, and "see" them on the hexagon in your mind.

Product Relationships
Any two NON-adjacent functions on the hexagon can be simplified by simply taking the function that is "between" them. "Between" for functions directly opposite each other means the "1" in the center, since functions on opposite sides of each other are reciprocals and the product of reciprocals is one. If functions are not directly opposite each other, then the function between them on the exterior of the hexagon is the simplification. Again, examples often make the idea more clear, so here are several:

Cos(x)Tan(x) = Sin(x)
Tan(x)Csc(x)= Sec(x)
Sin(x)Sec(x)= Tan(x)

Quotient Relationships
By reversing the product relations, we can create simple division relationships between any two functions which are next to each other on the Hexagon. To find the simplification of the quotient, think of the phrase "from the denominator, past the numerator, to the answer".

OK, not the simplest thing in the world; so if you come up with a better mneumonic for this, drop me a line. Use these examples to help you see how it works.

For Sin(x)/ Cos (x); start at the denominator, cos(x), and go in the direction of the numerator, sin(x), to the next function, tan(x) which is the simplification. Here are some other relations from the Hexagon:

Tan(x)/Sec(x)=Sin(x)
Sec(x)/Csc(x)=Tan(x)
Csc(x)/Sec(x)=Cot(x)
Cot(x)/Csc(x)=Cos(x)

Compare the first and the fourth relationship in this list to discover a nice relationship for identities. If you replace every term in an identity with its co-function, you get another true identity.