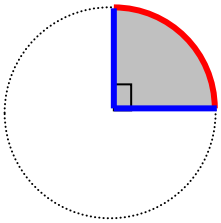
The Surface Area of a Cone



Consider the diagram alongside. The red line is an **arc** of the circle and the **radius** is blue. The shaded area is a **sector** of the circle. The dotted line is the circumference of the circle.

The following should be obvious:

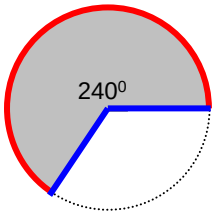
The area of the sector is ¼ of the area of the circle

The arc is ¼ of the circumference of the circle

Written as a proportion:

$$\frac{area\ of\ sector}{area\ of\ circle} = \frac{length\ of\ arc}{circumference\ of\ circle} = \frac{1}{4}$$

Don't read on until you understand why this is true!



Now there is nothing special about ¼. The diagram alongside shows that:

$$\frac{area\ of\ sector}{area\ of\ circle} = \frac{length\ of\ arc}{circumference\ of\ circle} = \frac{2}{3}$$

Generally,

For any sector of a circle:

$$\frac{area\ of\ sector}{area\ of\ circle} = \frac{length\ of\ arc}{circumference\ of\ circle}$$

We will use this fact to discover the formula for the area of the curved surface of a cone.

Small Group Activity The Surface Area of a Cone

Materials: For each group: a piece of paper, a compass, a length of sticky tape, a red marker and a blue marker.

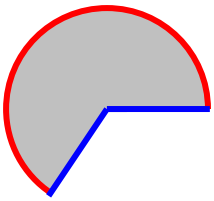
1. In this activity, you will be guided to the discovery of the formula for the area of the curved surface of a cone.
   a. Before you start this investigation, read the introduction to this section.
   b. Explain in your own words why, for any sector of any circle:

$$\frac{area\ of\ sector}{area\ of\ circle} = \frac{length\ of\ arc}{circumference\ of\ circle}$$

2. Draw a circle of radius 10 cm. Cut out the circle.

   If you cut out a sector of the circle, you will obtain the net of a cone. If you cut out a small sector, the cone will be tall and thin. If you cut out a large sector, the cone will be short and fat.

3. Decide on the shape of the cone you want, and cut out the sector. Now colour the arc remaining in **RED**.

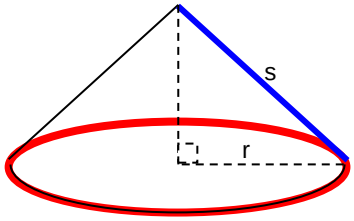


4. Tape the edges of the cut (no overlap!) to make your cone. What colour is the circumference of your cone? Copy and complete this sentence:

   I fold the net of a cone to make the cone. The arc of the net = _____

5. The distance from the apex of the cone to the circumference is called the **slant height**. Use the blue marker to draw in the slant height. Label it *s*.

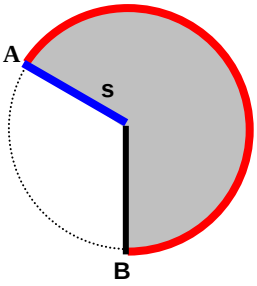
   You should now have a cone that looks similar to that shown alongside (it might be shorter or taller).



6. Write the formula for the circumference of the cone (shown in red) in terms of the radius of the base, *r*.

7. Cut the cone along the slant height.

   a. Write the formula for the length of the arc AB. (Hint: remember what it equals!)
   b. Write the formula for the circumference of the net with radius *s*, shown alongside.
   c. Explain why the area of the curved surface of the cone is equal to the area of the sector.



   To find the area of the curved surface, we find the area of the sector. From the previous section,

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{length of arc}}{\text{circumference of circle}}.$$

8. Use algebra to show that:

$$\text{area of sector} = \frac{\text{length of arc} \times \text{area of circle}}{\text{circumference of circle}}$$

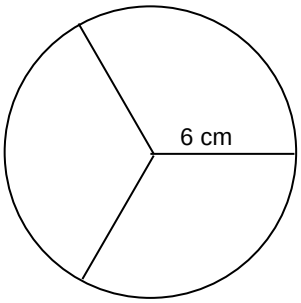
9. Substitute into the above formula, and divide out common factors. Hence show that the area of the sector (and the area of the curved surface of the cone) is given by:

$$A = \pi rs$$

   Congratulations! You have discovered that

> The formula for the surface area of the curved side of the cone is
>
> $$SA = \pi rs$$
>
> where SA is the surface area of the curved side of the cone
> r is the radius of the base of the cone
> s is the slant height of the cone.

Challenge Activity *Finding the Height of a Cone*



A circle of radius 6 centimetres is cut into three congruent sectors. Each sector is formed into a cone, with no overlap.

What is the height of each cone?

Note: This can be done without constructing the cones, using your knowledge of space, formulas for circumference, algebra and Pythagoras'Theorem!