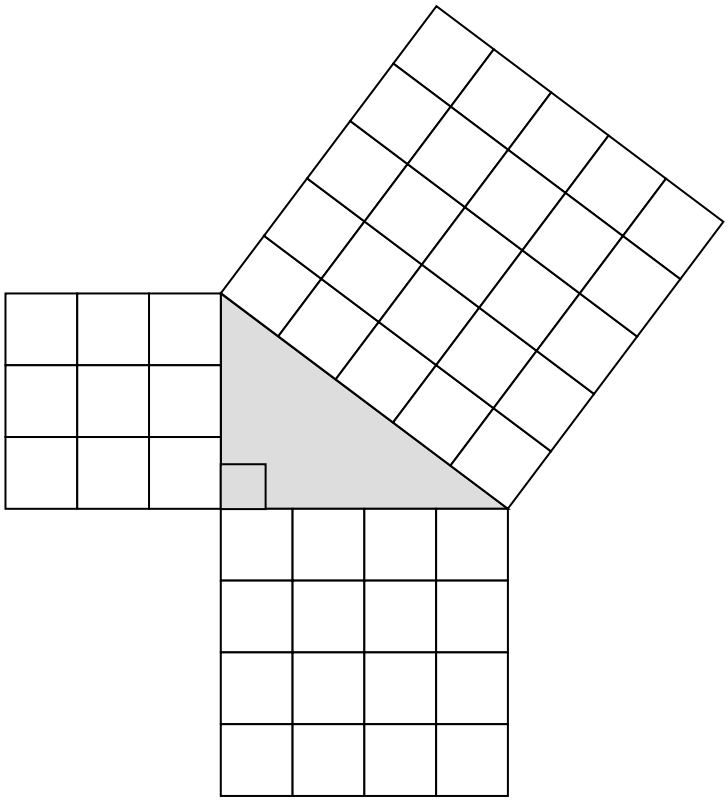
The Theorem of Pythagoras



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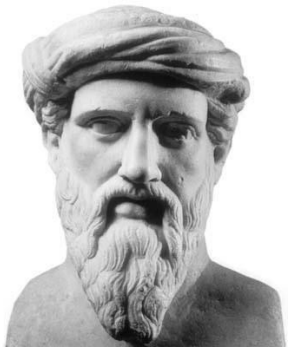
The Theorem of Pythagoras



Pythagoras was born in the island of Samos in ancient Greece. There is no certainty regarding the exact year when he was born, but it is believed that it was around 570 BC. If he was alive today, he would be about 2,575 years old!

Pythagoras visited Egypt around 547 BC when he was 23 years old. He stayed in Egypt for 21 years learning a variety of things including geometry from Egyptian priests. It was probably in Egypt where he learned the theorem that is now called by his name. The Egyptians used his theorem to design the Great Pyramids.

When he was 55 years old he moved to Croton in the south of Italy, where he started a school which concentrated in the teaching and learning of Mathematics, Music, Philosophy, and Astronomy and their relationship with Religion. Pythagoras also headed a cult known as the secret brotherhood that worshiped numbers and numerical relationships. They attempted to find mathematical explanations for music, the gods, the cosmos – in fact for everything! Pythagoras believed that all relations could be reduced to number relations. He died at age 99.

Squares and Square Roots

1. To understand and use the Pythagorean Theorem, it is useful to know the squares of numbers from 1 to 20. **Complete this table**:

$1^2 = 1$	$2^2 = 4$	$3^2 =$	$4^2 =$	$5^2 =$
$6^2 =$	$7^2 =$	$8^2 =$	$9^2 =$	$10^2 =$
$11^2 =$	$12^2 =$	$13^2 =$	$14^2 =$	$15^2 =$
$16^2 =$	$17^2 =$	$18^2 =$	$19^2 =$	$20^2 =$

2. The numbers 1, 4, 9, 16, … are called square numbers since each is the square of a whole number. The inverse of squaring is taking a square root. That is, $\sqrt{1} = 1$, $\sqrt{4} = 2$, … $\sqrt{225} = 15$. **Complete this table**:

$\sqrt{1} = 1$	$\sqrt{4} = 2$	$\sqrt{9} =$	$\sqrt{16} =$	

3. Use a calculator to find the square root of each of the following. Round your answer to the number of decimal places given in the brackets.

a. $\sqrt{3}$ (1) _____ b. $\sqrt{5}$ (2) _____

c. $\sqrt{19}$ (1) _____ d. $\sqrt{146}$ (3) _____

e. $\sqrt{7}$ (6) _____ f. $\sqrt{529}$ (2) _____

4. a. In the space below, use a ruler to draw a number line from 0 to 10. Make the number line 10 cm long.

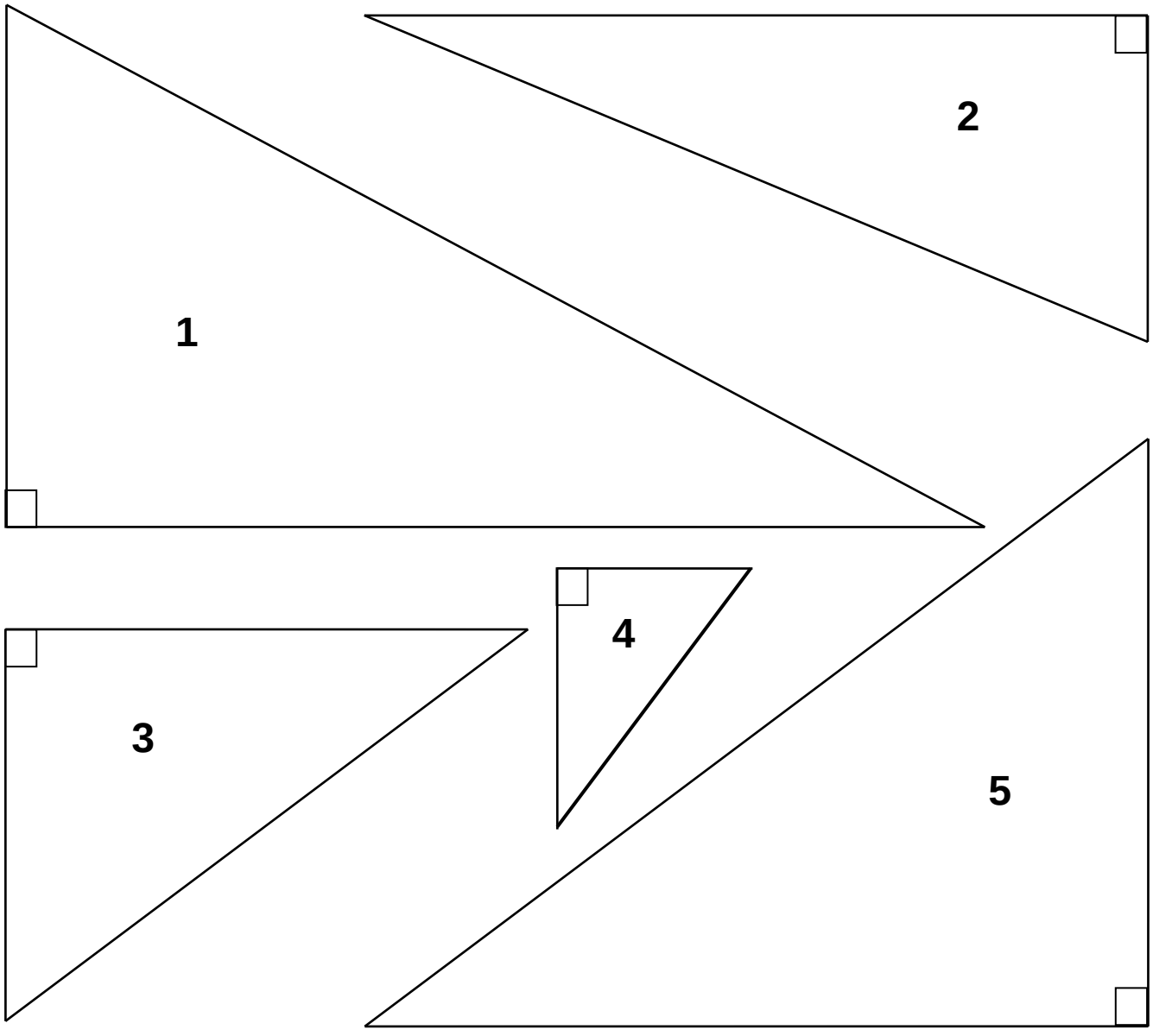
b. Mark as accurately as you can the following numbers on the number line:

i. $\sqrt{3}$ ii. $\sqrt{5}$ c. $\sqrt{30}$ d. $\sqrt{90}$

5. For each of these right-angled triangles, use your centimeter ruler to measure all three sides. Call the shortest side A, the next shortest side B, and the longest side C. Record their lengths in the table below.

Triangle No.	Side A	Side B	Side C	$(\text{Side A})^2$	$(\text{Side B})^2$	$(\text{Side C})^2$
1						
2						
3						
4						
5						

Complete all of the columns. Look for a pattern. Find a rule for the lengths of the sides of a right-angled triangle. For example, what is the rule for finding the length of the longest side, if you know that the lengths of the two shortest sides of a triangle are 7 cm and 24 cm?



List any patterns you discovered: ……

…….....

One of the patterns can be used to find the length of the hypotenuse if you know the length of the other two sides. Write that pattern here: …….....

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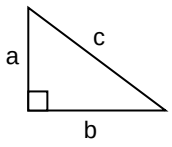
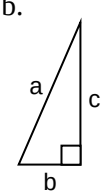
6. In a right triangle, the side opposite the right angle is called the **hypotenuse**. It is always the longest side.

In each diagram below, identify the hypotenuse.

a.



b.



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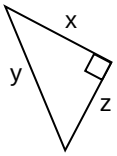

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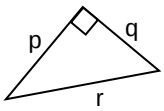
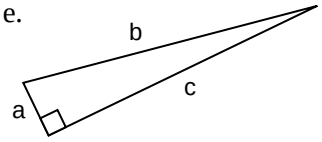


e.



7. Explain clearly how the diagram on the right verifies the Theorem of Pythagoras. Write in full sentences.

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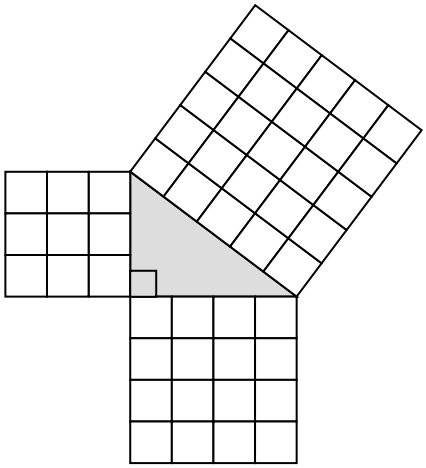
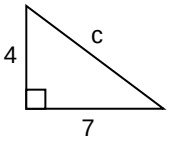


Example 1

Use the Pythagorean Theorem to find the length of the hypotenuse, to 1 decimal place.



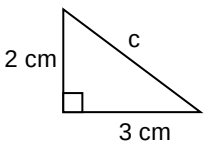
Solution

From the Pythagorean Theorem

	$c^2 = a^2 + b^2$	{Write the Pythagorean Theorem}
	$c^2 = 4^2 + 7^2$	{Substitute}
	$c^2 = 16 + 49$	{Square both numbers}
	$c^2 = 65$	{Simpify}
$\therefore$	$c = \sqrt{65}$	{Take the square root of both sides. This is the **exact solution.**}
	$c = 8.1$	{Round to 1 decimal place. This is an **approximate solution**.}

8. Find the length of the hypotenuse. Round to 1 decimal place, where appropriate. **Set out your work as above. Answer in your homework book.**

a.



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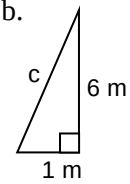

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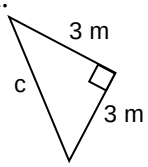

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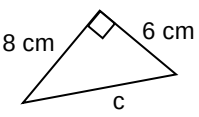

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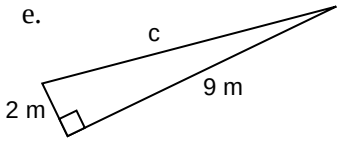
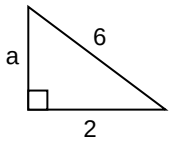


Example 2

Use the Pythagorean Theorem to find the length of the short side, rounded to 2 decimal places.



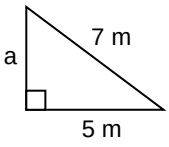
Solution

From the Theorem of Pythagoras:

	$a^2 = c^2 - b^2$	{Write the Pythagorean Theorem}
	$a^2 = 6^2 - 2^2$	{Substitute}
	$a^2 = 36 - 4$	{Square both numbers}
	$a^2 = 32$	{Simplify}
$\therefore$	$a = \sqrt{32}$	{Take the square root of both sides. This is the **exact solution.**}
	$a = 5.66$	{Round to 1 decimal place. This is an **approximate solution**.}

9. Find the length of the unknown short side. Round to 2 decimal places, where appropriate. **Set out your work as above. Answer in your homework book.**

a.



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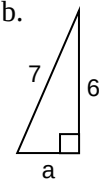

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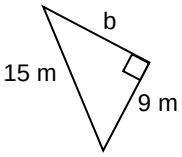

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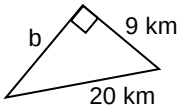

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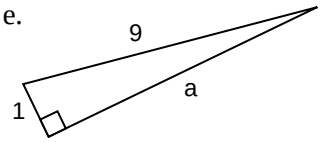
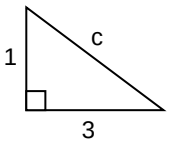
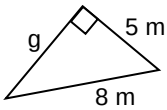


10. Find the length of the unknown side. Round to 1 decimal place where appropriate. **Set out your work as above. Answer in your notebook.**

a.



b.



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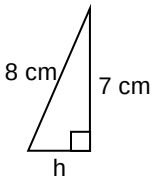

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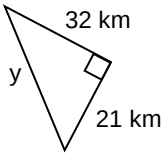

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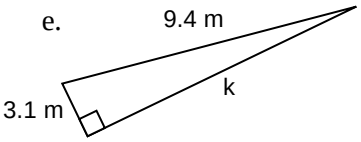
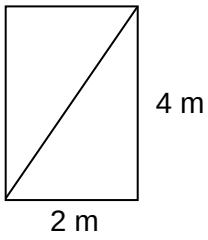


Applications of the Pythagorean Theorem

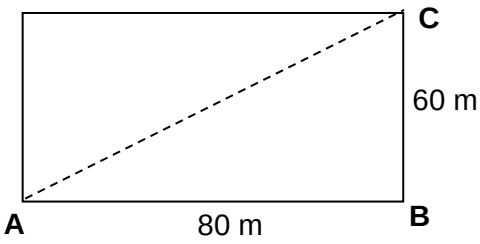
Solve the following word problems. For each problem, draw a neat diagram. You may have to draw additional lines to make a right triangle. **Set your work out correctly. Answer in your homework book.**

1. I am making a rectangular frame that is 4 metres high and 2 metres wide. I need to brace it with a diagonal brace. How long is the brace?

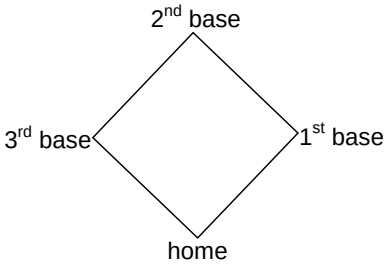


2. Kyle and Alesha have to walk from one corner of a park to the diagonally opposite corner (from A to C in the diagram).

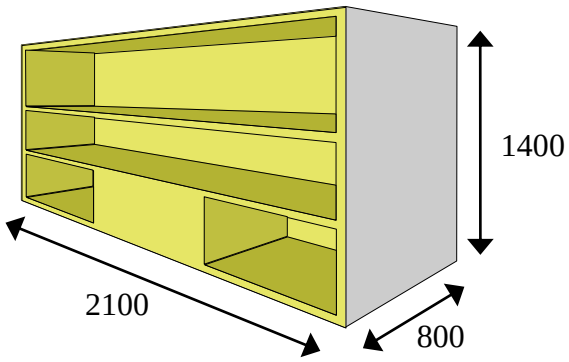
   How much shorter is it to walk diagonally across the park (directly from A to C) than to walk around it (from A to B to C)?



3. The distances between bases in a softball field is 60 feet (see diagram). How far it is from home to 2nd base?



4. You are building the bookcase at the right. To finish it, you want to put a diagonal brace across the back. How long should the brace be?



For the following questions, a diagram isn't given. Draw the diagram first.

5. I have a ladder that is 10 metres long. I lean it against a wall. The foot of the ladder is 3 metres from the base of the wall.
   i. Draw a neat diagram of this situation. Label the diagram. Let h = height that the ladder reaches up the wall.
   ii. Use the Pythagorean Theorem to find h, to the nearest centimetre.

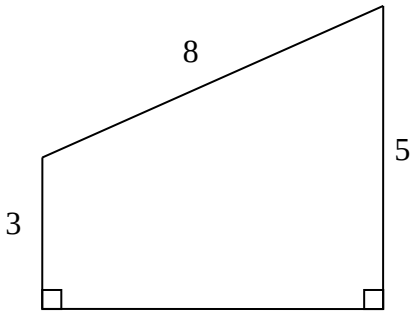
6. Tony has got his kite stuck at the top of a very tall tree. He knows the string on his kite is 20 m long. When he pulls the string tight and holds the very end on the ground it touches 17 m from the bottom of the tree. If the ground is flat, how tall is the tree?

7. The guy wires holding up an 11 m radio transmitter are attached 2 m from the top of the transmitter, and are anchored to the ground 7 m from the base of the aerial. How long are the guy wires?

Challenge Problems!

1. a. Sketch the shape alongside. Add a line to make a right-angled triangle.

   b. Use Pythagoras' Theorem to find the value of *y* in the diagram alongside.

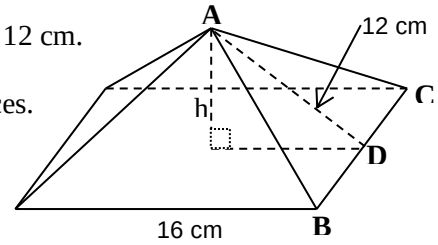


2. The square-based pyramid on the right has a base edge length of 16 cm. Each of the triangular faces has a height of 12 cm, ie, AD has length 12 cm.

   a. Draw only the right-angled triangle in the diagram. Label all vertices.

   b. Use Pythagoras' Theorem to find the perpendicular height, *h*.

   c. Find the volume of this square-based pyramid.



3. a. Sketch the shape alongside. Add a line to make a right-angled triangle.

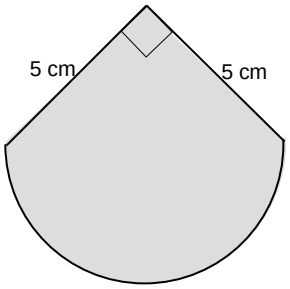
   b. Find the area of the right-angled triangle.

   c. Find the length of the hypotenuse of the right-angled triangle.

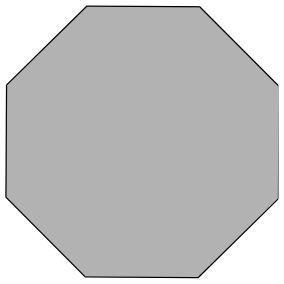
   d. Find the radius of the semicircle.

   e. Find the area of the semicircle.

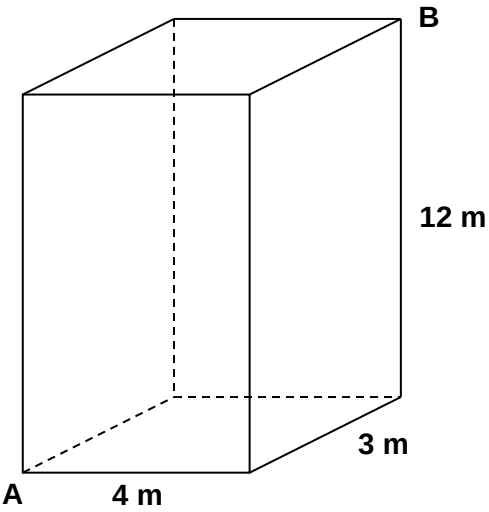
   f. Find the area of the shape



4*. Calculate the area of the regular octahedron on the right.



5*. Calculate the distance from A to B in the diagram alongside.



Answers – Applications of the Pythagorean Theorem

1. 4.472 m 2. 40 m 3. 84.85 ft 4. 2523 mm 5. 9.54 m

6. 10.54 m 7. 11.40 m

Answers – Challenge Problems

1. 7.746 m 2. h = 8.944, V = 763 cm^3

3. b. 12.5 cm^2 c. 7.07 cm d. 3.54 cm e. 19.6 cm^2

   f. 32.1 cm^2

4. 43.46 cm^2 5. 13 m