Teaching Tip - The Only Area Formula You Will Ever Need

Middle school mathematics consists of a small number of simple ideas. Often math teachers, trying to make math simple by giving a rule for everything, actually make it more complex.

Area is an example of this. Consider these area formulas, commonly taught in middle school and high school:

Area = side^2	{square}
Area = length x width	{rectangle}
Area =base x perpendicular height	{parallelogram}
Area = $\frac{1}{2}$ x base x perpendicular height	{triangle}
Area = π x radius^2	{circle}
Area = π x major axis x minor axis	{ellipse}
Area = $\frac{1}{2}$ x perpendicular height x (base1 + base2)	{trapezoid}

The use of different terminology for the same dimensions implies that these area formulas are unrelated. So not only do students have to memorize seven formulas, and to remember which formula to use for each shape, but they miss the big picture. Hey, area is area. How much easier and how much more logical it would be if there was only one area formula to learn!

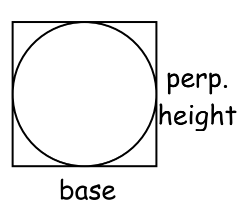
Fortunately there IS only one area formula:

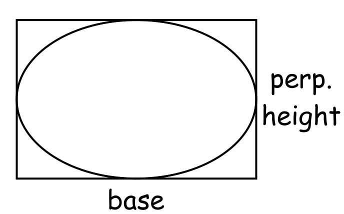
Area = Modifier x Base x Perpendicular Height

Here are the above formulas, re-written using this one formula:

Area = base x perpendicular height	{square, modifier is 1}
Area = base x perpendicular height	{rectangle, modifier is 1}
Area = base x perpendicular height	{parallelogram, modifier is 1}
Area = $\frac{1}{2}$ x base x perpendicular height	{triangle, modifier is $\frac{1}{2}$}
Area = $\frac{3}{4}$ x base x perpendicular height	{circle, modifier is $\frac{3}{4}$}
Area = $\frac{3}{4}$ x base x perpendicular height	{ellipse, modifier is $\frac{3}{4}$}
Area = average base x perpendicular height	{trapezoid, modifier is 1}

Actually, I cheated (a bit). The formulas for the area of a circle and an ellipse are only approximate. However, if you look at the diagrams below, you will see that they are reasonable estimates. The area of the circle and the ellipse are both about $\frac{3}{4}$ of the area of the surrounding rectangle.





If you want a more accurate area, just replace the 3 with π:

Area = $\frac{\pi}{4}$ x base x perpendicular height {circle}

Area = $\frac{\pi}{4}$ x base x perpendicular height {ellipse}

The trapezoid presents a bit of a puzzle – it has two bases, so which do we use? Well, one is too short, the other is too long. So, just find the average of the bases and use that!

Some explanatory notes on my area formula

1. When I write the formula, I write the words "base" and "perpendicular height", mainly to cement in the idea that the height has to be measured perpendicular to the base. After a while I will save writing by using the variables *b* and *h*. But even though I write the letters, I continue to *say* "base" and "perpendicular height".

2. The area of the triangle is given as
   Area = $\frac{1}{2}$ x base x perpendicular height

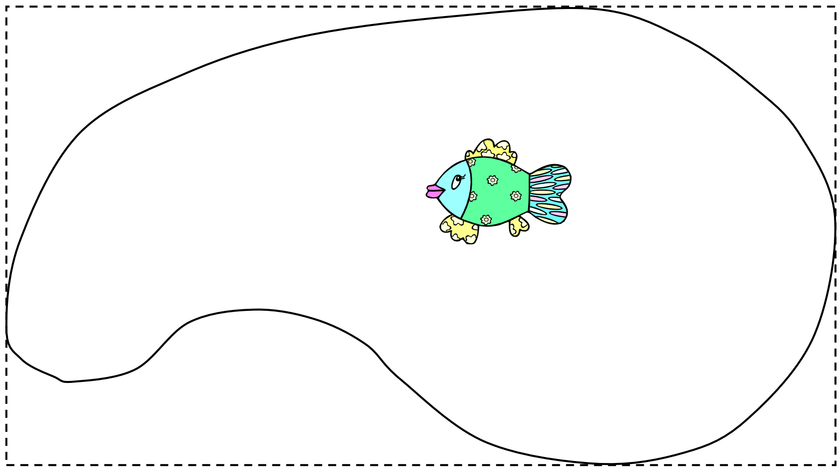
   I prefer the following equivalent formula for the area of a triangle:
   Area = base x perpendicular height ÷ 2
   It is easier to divide by 2 than to multiply by $\frac{1}{2}$.

3. My formula for the area of a circle is different to the usual $A = \pi r^2$. This causes some consternation amongst math teachers (and the usual formula may be on an external test somewhere) so maybe we should also derive the usual formula:

Area = $\frac{\pi}{4}$ x base x perpendicular height	
Area = $\frac{\pi}{4}$ x diameter x diameter	{since base = p. height = diameter}
Area = $\frac{\pi}{4}$ x 2 x radius x 2 x radius	{since diameter = 2 x radius}
Area = $\frac{\pi}{4}$ x 4 x radius x radius	{re-arranging}
Area = π x radius x radius	{dividing out the common factor}
Area = πr^2	{Though I prefer Area = π x r x r}

Areas of odd shapes

In real life, many shapes are not geometric. How do we find the area of odd shapes? Here is a diagram of a fishpond (with its surrounding rectangle). What is its area?



Our area formula works here too!

area = modifier x base x perpendicular height

What is the modifier in this problem? We will have to estimate it. My guess is that the fish pond occupies about ¾ of the surrounding rectangle. So…

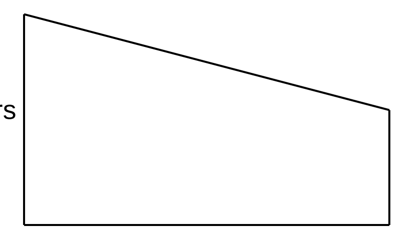
area = ¾ of 12 x 8

= 9 x 8 = 72 feet^2

And for real life purposes, that is close enough!

Area of compound shapes

What about shapes that are composed of two or more geometric shapes? Consider the shape alongside.



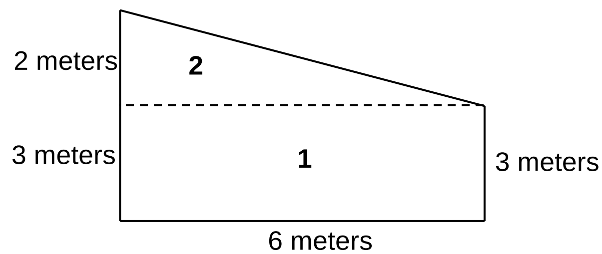
We have 2 methods we can use.

1. We could surround it with a rectangle and estimate the modifier. This can be done in one's head. Think:

   5 x 6 = 30; modifier is about ¾.

   Now ¼ of 30 = 7½, so ¾ of 30 = 22½ meter^2.

2. If I need a more accurate answer, I will use the traditional method of breaking the compound shape into two known shapes.



$Area_1 = b \times h = 3 \times 6 = 18\ m^2$
$Area_2 = b \times h \div 2 = 6 \times 2 \div 2 = 6\ m^2$
$Area = Area_1 + Area_2 = 18 + 6 = 24 m^2$

Teaching Sequence

Here is my preferred order for teaching area.

1.	Area of a rectangle	Start by drawing rectangles on graph paper.
2.	Area of odd shapes	This introduces the idea of the "modifier".
3.	Area of parallelogram	Cut it into two pieces and make a rectangle.
4.	Area of triangle	Show that two triangles make a parallelogram, so one triangle is half the area of the parallelogram
5.	circle	As done previously. I also show the standard "proof" of the formula by cutting the circle into sectors and re-arranging to make a parallelogram.
6.	trapezoid	I don't teach the formula. I prefer to treat this as a compound shape. Every trapezoid can be split into a parallelogram and a triangle, so we don't need to treat the trapezoid as a special case.