

Investigation: Linear Equations and Sequences

Linear equations can be written in the form: $ax + by = c$. Some rather surprising results can occur when the values of a , b and c are not arbitrary, but conform to some rule. A graphics calculator can be useful for discovering the pattern, and algebra can then be used to confirm (and sometimes explain) the pattern.

1. Consider linear equations of the form $ax + by = c$, where a , b and c form an arithmetic sequence. Some examples: $2x + 4y = 6$, and $3x - y = -5$
 - a. Construct a number of such linear equations, and graph them using a graphics calculator. You will discover something surprising.
 - b. Describe what you have found using the language of mathematics.
 - c. Give an argument, using algebra, explaining why this pattern must hold.

2. Consider linear equations of the form $ax + by = c$, where a , b and c form a geometric sequence. Some examples: $2x + 4y = 8$, and $-x + 2y = -4$
 - a. Construct a number of such linear equations, and graph them using a graphics calculator. You will need to graph quite a few lines before the pattern emerges.
 - b. Describe the pattern using the language of mathematics.
 - c. Use algebra to explain why this pattern must hold (NB: this requires sophisticated analysis – i.e. it is a tough question. See your teacher if you need hints).

- 3a. Make up your own rule that restricts the values of a , b and c .
 - b. Use a graphics calculator to graph some lines that conform to your rule.
 - c. If an interesting pattern emerges, describe the pattern using the language of mathematics. Then use algebra to explain why the pattern must hold.
 - d. If no interesting pattern emerges, then record your attempt for posterity, and try a different rule.