

Staircase Numbers

What are Staircase Numbers?

Staircase numbers are numbers that can be written as a sum of consecutive counting numbers (ie, positive whole numbers not including zero).

Some Examples:

5 is a staircase number, as it can be written as $2 + 3$.

12 is a staircase number because it can be written as $3 + 4 + 5$

15 can be written as $7 + 8$, or as $4 + 5 + 6$, or as $1 + 2 + 3 + 4 + 5$.

Not all numbers are staircase numbers; for example 2 is not a staircase number. '

Ten Questions - Prove your answers where possible.

1. Which numbers can't be written as staircase numbers?
2. Which staircase numbers can be written as the sum of exactly two consecutive numbers?
3. Which staircase numbers can be written as the sum of exactly three consecutive numbers?
4. Which staircase numbers can be written as the sum of exactly four consecutive numbers?
5. Which numbers can only be written as a staircase number in one way?
6. Find all of the ways that 105 can be written as a staircase number.
7. Find all of the ways that 2001 can be written as a staircase number.
8. Find all of the ways that 2002 can be written as a staircase number.
- 9*. An algorithm is a step-by-step procedure for doing a given task. Write an algorithm that shows how to write each staircase number as a sum of consecutive integers in all possible ways.
- 10*. Some numbers can be written as the sum of consecutive even numbers,

eg. $12 = 2 + 4 + 6$. Some numbers can be written as the sum of consecutive odd numbers, eg. $24 = 3 + 5 + 7 + 9$. Which numbers can't be written in either of those two forms?