

## The most amazing number - 142857

I have a most unusual pet. It's not a dog. It's not a cat. It's a number. It's a most amazing number, and it does tricks much more interesting than rolling over, or playing dead. My pet number is: **142857**.

Big deal, you say? What kind of 'tricks' can a number like that do? Good question! Let me show you the first trick my pet number can do. Watch what happens when you multiply it by 2.

$$142857 \times 2 = 285714$$

Study those numbers carefully. What do you notice? Hopefully you noticed that both numbers contain the same digits: 1,2,4,5,7, and 8.

If you were looking carefully, though, you may have noticed something truly amazing: not only does the second number use the same digits, it uses them *in the same order!*

Okay, so that's mildly amusing...is that the only trick it does? Oh, no! We're just getting started!

$$142857 \times 3 = 428571$$

$$142857 \times 4 = 571428$$

$$142857 \times 5 = 714285$$

$$142857 \times 6 = 857142$$

In each case, the result of the multiplication uses the *same* digits, in the same order as the original number.

But why stop at six? Why not multiply by seven? Okay...

$$142857 \times 7 = 999999$$

Amazingly, the tricks don't stop there, either. Watch what happens when you multiply by eight...

$$142857 \times 8 = 1142856$$

This doesn't look exactly like the original number, but if you take the first digit off the front of the answer, and add it to the last six digits, look what happens:

$$1 + 142856 = 142857$$

You can do this with any number you choose. Watch what happens when you multiply my pet number by 326:

$$142857 \times 326 = 46571382, \text{ and } 46 + 571382 = 571428$$

The only exception to this pattern is the multiples of seven:

$$142857 \times 266 = 37999962, \text{ and } 37 + 999962 = 999999$$

Here are some interesting things to try:

- Can you find a number that doesn't do anything interesting when multiplied by my pet number?
- What is the decimal value of the fraction one-seventh? two-sevenths? etc.
- Can you find any numbers that behave the same way as 142857?

For those who have some computer programming skill and knowledge of number theory, finding numbers that behave like 142857 is a good project. A clue that may help the student get started is to notice that

$$1/7 = .142857142857142857\dots$$

I've found several numbers like this. One of them is so large it takes an entire sheet of paper to print it! In fact, it's so large that it contains *every* four digit telephone number!