# Learning Mensuration Formulae

In the upper primary and lower secondary years, students are expected to learn to find perimeters of polygons, circumference of circles, areas of rectangles, triangles and circles and volumes and surface area of prisms, cylinders, pyramids, cones and spheres. Some of these can be derived by common sense, but for others, some memorisation of formulae is necessary. Even by the end of year 10, many students do not manage to learn all the required formulae.

There are a few ways of helping students to remember the formulae.

The firstly is the idea of dimensions. Area (including surface area) is a two-dimensional quantity and any area formula will involve the product of two lengths measured at right angles to one another. If these lengths are $a_1$ and $a_2$, then the area formula will be of the form $a_1 \times a_2 \times k$, where $k$ is a constant. This idea can be introduced when students meet the area of a rectangle. The idea of the measurements being at right angles will help students to grasp the methods for finding areas of triangles and, where appropriate, parallelograms and ellipses.

Volume is a three-dimensional quantity and any volume formula will involve the product of three lengths each measured at right angles to the other two. If these lengths are $a_1$, $a_2$ and $a_3$, then the volume formula will be of the form $a_1 \times a_2 \times a_3 \times k$, where $k$ is a constant.

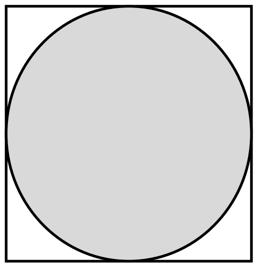
Perimeter is a one-dimensional quantity and any perimeter formula will contain length measurements, but no products of length measurements. It will be of the form $a_1 \times k_1 + a_2 \times k_2 + \ldots$

If students can internalise this, they should have fewer problems confusing formulae, for instance recalling the circumference of a circle as $\pi r^2$ or the area of a rectangle as $2 \times (l + b)$.

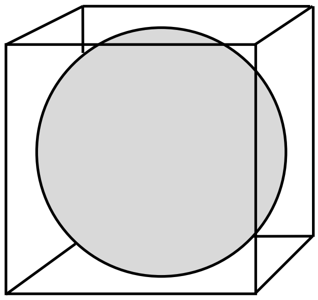
In most cases, the choices for $a_1$, $a_2$ etc. are fairly obvious. What is less obvious is the value for $k$, especially for circles and spheres. But there are some strategies for learning these.

Firstly anything round has a π in it.

Secondly, circle and sphere formulae in terms of diameter have a more consistent form than the formulae in terms of radius, especially when they are viewed in terms of the enclosing square or cube.



If asked what fraction of the enclosing square is occupied a circle, students will generally estimate around the $^3/_4$ mark. From this, students can calculate the area of a circle as $^3/_4$ that of the enclosing square. In terms of $a_1 \times a_2 \times k$, this is diameter × diameter × $^3/_4$. Similarly, if told that the perimeter of the enclosing square is 1m, students will estimate that the perimeter of the circle is about ¾ m. Thus the area and circumference of a circle are both ¾ of that of the enclosing square. Later, students can be introduced to the idea that 3 quarters is an approximate value: 3.14... quarters is the more exact value.



In the same way students can be asked what fraction of the volume of a cube is taken up by the enclosed sphere. After recording some estimates, the value can be checked by immersing a sphere in a cubic container of water. The fraction obtained is about $^1/_2$. This will do for most practical purposes, but if the displacement experiment is performed carefully, it can be seen that the fraction is slightly more than $^1/_2$. With a suggestion that a π is involved, many students will guess that the actual fraction is $^\pi/_6$. The surface area of a sphere is more difficult to determine experimentally, but it also comes to $^\pi/_6$ of the surface area of the enclosing cube.

Thus for circles, both perimeter and area are π quarters of that of the enclosing square; for spheres, both surface area and volume are π sixths of that of the enclosing cube. This is probably easier to remember than the traditional formulae in terms of radius. Admittedly, the calculation is slightly longer because the enclosing square or cube has to be determined first, but, for most students, the increase in obviousness and reliability is probably worth the extra few seconds.