Three problems

1. For x, y > 0, prove $\frac{x}{y}+\frac{y}{x}\geq 2$

SOLUTION

Without loss of generality, assume that x ≥ y

Now $\frac{x}{y}+\frac{y}{x}=\frac{x^2+y^2}{xy}$

Since x ≥ y

$$\frac{x^2+y^2}{xy}\geq\frac{y^2+y^2}{yy}$$

$$\geq\frac{2y^2}{y^2}$$

$$\geq 2$$

2. Prove log 5 is irrational.

SOLUTION

Students needed only to know the definition of a logarithm:

$\log_b x = y$ implies $b^y = x$.

The key is indirect proof: Assume log 5 is rational, then log 5 = a/b, where a and b are integers. Consequently 10^(a/b) = 5 or 10^a = 5^b.

This is not possible, since all postive powers of 10 end in 0, while all positive powers of 5 end in 5.

Hence the assumption that log 5 is rational is not correct.

3. Given two real numbers a and b, write an expression (in terms of a and b) equal to the smaller of the two numbers.

SOLUTION

min(a, b) =(a+b)/2 - |a-b|/2.

In words, the smaller number is their average minus half the difference.