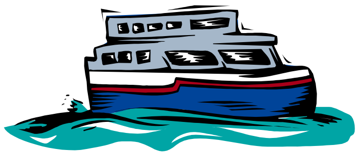
Knowledge & Procedures

1. A cannonball is fired upwards at 80 m/s. Gravity causes it to accelerate at 9.8 m/s downwards. Write and solve differential equations to find the relation between height and time and hence find how long it takes to come back down. (Use of $s = ut + \frac{1}{2}at^2$ or similar equations is not acceptable.)
   What assumptions need to be made in arriving at your answer?

2. A hovercraft is moving horizontally at 8 m/s. It is acted upon by air resistance. The acceleration (in m/s^2) produced by the air resistance is equal to –0.12 times the velocity (in m/s). Write and solve differential equations to find the relation between distance travelled and time and hence find how far it travels before it stops.



3. A cannonball is fired upwards at 80 m/s. Gravity causes an acceleration of 9.8 m/s^2 downwards. Air resistance produces an acceleration (in m/s^2) equal to –0.05 times the velocity (in m/s). Write and solve differential equations to find the relation between height and time and hence find how long it takes to come back down.

Modelling & Problem Solving

4. An artillery shell is fired at 120 m/s in a direction 50° above the horizontal. Gravity causes an acceleration of 9.8 m/s^2 downwards. Air resistance produces an acceleration (in m/s^2) equal to –0.05 times the velocity (in m/s). By writing and solving differential equations, find how far away the shell lands.
   What assumptions are necessary in arriving at your answer?

5. Go to Scholaris, open the Excel spreadsheet called 'C12 Sem 2 Assig 2012 SS.xlsx' and look at the sheet named 'Cannonball 1'. It solves the problem in Question 1 numerically. Explain how it does this.
   What are the strengths and limitations of this method?
   Justify the reasonableness of the result using your result from Q1.
   How could the model used be refined to produce a more accurate result?

6. Look at the sheet named 'Cannonball 3'. This is a copy of 'Cannonball 1'. Add a column for acceleration between the time and velocity columns; include a cell for the coefficient of air resistance in Row 13; and make any other necessary modifications so that this sheet solves the problem in Question 3.
   Justify the reasonableness of your result using your result from Q3.
   By adjusting the value for the coefficient of air resistance, find the coefficient required for the cannonball to spend just 10 seconds in the air.

7. Use the blank sheet named 'Artillery Shell' to produce a spreadsheet that will plot the path of the shell in Question 4 and hence find how far away the shell lands.

Your completed spreadsheet must be emailed to d.ilsley@canterbury.qld.edu.au by the due time for the assignment.

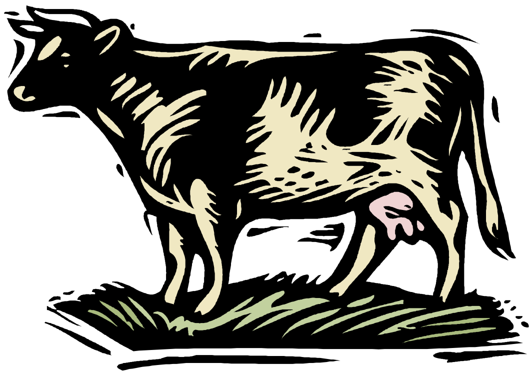
Questions 8 and 9 will be completed under examination conditions immediately after Questions 1 to 7 are handed in.

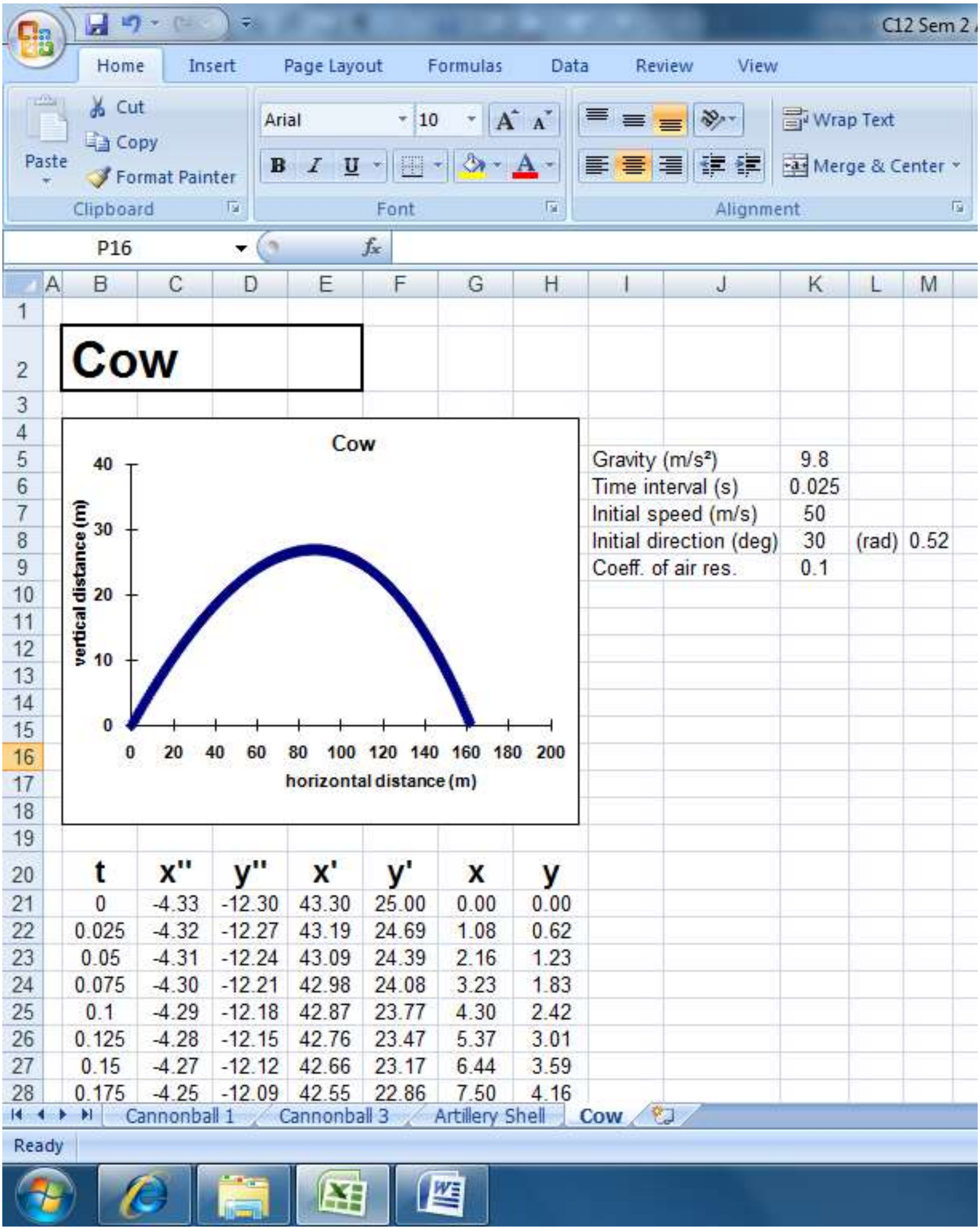
Questions 8 and 9 to be completed under examination conditions immediately after Questions 1 to 7 are handed in.

Knowledge & Procedures

8. A cow is fired from a catapult at 50 m/s in a direction 30° above the horizontal. Gravity causes an acceleration of 9.8 m/s^2 downwards. Air resistance produces an acceleration (in m/s^2) equal to –0.1 times the velocity (in m/s). Assume that the ground is flat and level. Write and solve differential equations to show that the cow lands just over 160 m from the catapult.

9. On the reverse of this page is part of a spreadsheet designed to plot the path of the cow in Question 8. Assuming that rows below Row 22 were filled down from Row 22, give likely formulae for cells B25 to H25. No justification is necessary.





Gravity (m/s²)	9.8		
Time interval (s)	0.025		
Initial speed (m/s)	50		
Initial direction (deg)	30	(rad)	0.52
Coeff. of air res.	0.1		

t	x''	y''	x'	y'	x	y
0	-4.33	-12.30	43.30	25.00	0.00	0.00
0.025	-4.32	-12.27	43.19	24.69	1.08	0.62
0.05	-4.31	-12.24	43.09	24.39	2.16	1.23
0.075	-4.30	-12.21	42.98	24.08	3.23	1.83
0.1	-4.29	-12.18	42.87	23.77	4.30	2.42
0.125	-4.28	-12.15	42.76	23.47	5.37	3.01
0.15	-4.27	-12.12	42.66	23.17	6.44	3.59
0.175	-4.25	-12.09	42.55	22.86	7.50	4.16

Year 12 Maths C Semester 2 Assignment Solutions

Q1.

$$\frac{dv}{dt} = -9.8$$

$$v = \int -9.8\, dt$$

$v = -9.8t + c$ $\quad$ *and since* $v = 80$ *at* $t = 0 \quad c = 80$

$v(t) = -9.8t + 80$

$\frac{dh}{dt} = -9.8t + 80$ $\quad$ *where h is height*

$$h = \int -9.8t + 80\, dt$$

$h = 80t - 4.9t^2 + c$ $\quad$ *and since* $h = 0$ *at* $t = 0 \quad c = 0$

$h(t) = 80t - 4.9t^2$

Time for projectile to land is when the height is next equal to 0

solve $h(t) = 0$

$t = 0$ (*initial*) $\quad$ *or* $t = 16.33s$ (*when it lands*)

Assumption is that there is no air resistance

Q2.

$$a = \frac{dv}{dt}$$

$$\therefore\ -0.12v = \frac{dv}{dt}$$

$$-0.12 \int dt = \int \frac{dv}{v}$$

$$-0.12t + c = \ln v$$

$$v = e^{-0.12t+c}$$

$$v = Ae^{-0.12t} \qquad where\ A = e^{c}$$

$$at\ t = 0\ v = 8 \quad therefore\ A = 8$$

$$v(t) = 8e^{-0.12t}$$

$$\frac{dx}{dt} = 8e^{-0.12t}$$

$$x = \int 8e^{-0.12t}\,dt$$

$$x = -66.67e^{-0.12t} + c$$

$$at\ t = 0\ x = 0$$

$$\therefore c = 66.67$$

$$then\ x(t) = 66.67e^{-0.12t} + 66.67$$

$$as\ t \to \infty \quad x \to 66.67m$$

Therefore the hovercraft travels 66.67m before it stops

Q3.

$$\frac{dv}{dt} = -9.8 - 0.05v$$

$$\int \frac{dv}{9.8 + 0.05v} = -\int dt$$

$$20\ln(9.8 + 0.05v) = -t + c$$

$$e^{-\frac{t}{20}}e^{c} = 9.8 + 0.05v$$

$$at\ t = 0\ v = 80$$

$$then\ e^{c} = 13.8$$

$$\therefore 0.05v = 13.8e^{-\frac{t}{20}} - 9.8$$

$$v(t) = 276e^{-\frac{t}{20}} - 196$$

$$\frac{dh}{dt} = 276e^{-\frac{t}{20}} - 196 \quad where\ h\ is\ height$$

$$h = \int 276e^{-\frac{t}{20}} - 196\ dt$$

$$h = -5520e^{-\frac{t}{20}} - 196t + k$$

$$at\ t = 0, h = 0$$

$$then\ k = 5520$$

$$\therefore h(t) = -5520e^{-\frac{t}{20}} - 196t + 5520$$

Solve for t when h next equals zero

$$t = 0\ (initial) \quad or\ t = 14.57s$$

Q4.
Needs to be split into horizontal and vertical components as acceleration and velocity are different.

Vertical

$$\frac{dv}{dt} = -9.8 - 0.05v \quad \text{where } v \text{ is vertical velocity}$$

$$\int \frac{dv}{9.8 + 0.05v} = -\int dt$$

$$20\ln(9.8 + 0.05v) = -t + c$$

$$e^{-\frac{t}{20}}e^{c} = 9.8 + 0.05v$$

$$at\ t = 0\ v = 120sin50$$

$$9.8 + 6sin50 = e^{c} \quad (\approx 14.3965)$$

$$then\ v(t) = \frac{14.3965e^{-\frac{t}{20}} - 9.8}{0.05}$$

$$v_V(t) = 287.93e^{-\frac{t}{20}} - 196$$

Hits ground when $x_V(t) = 0$

$$x_V = -5759e^{-\frac{t}{20}} - 196t + 5759$$

$$solve\ for\ x_V(t) = 0$$

$$t = 0\ (initial) \quad or\ t = 16.52s\ (next\ time)$$

Horizontal

$$\frac{du}{dt} = -0.05u \qquad \text{where } u \text{ is horizontal velocity}$$

$$20\ln u = -t + c$$

$$u = Ae^{-\frac{t}{20}}$$

$$at\ t = 0\ u = 120cos50$$

$$A = 77.13$$

$$u = 77.13e^{-\frac{t}{20}}$$

$$x_H(t) = -1542.6e^{-\frac{t}{20}} + \mathrm{k}$$

$$at\ t = 0\ x = 0 \quad \therefore c = 1542.6$$

$$x_H(t) = -1542.6e^{-\frac{t}{20}} + 1542.6$$

Horizontal distance travelled – sub in 16.52s into $x_H(t)$

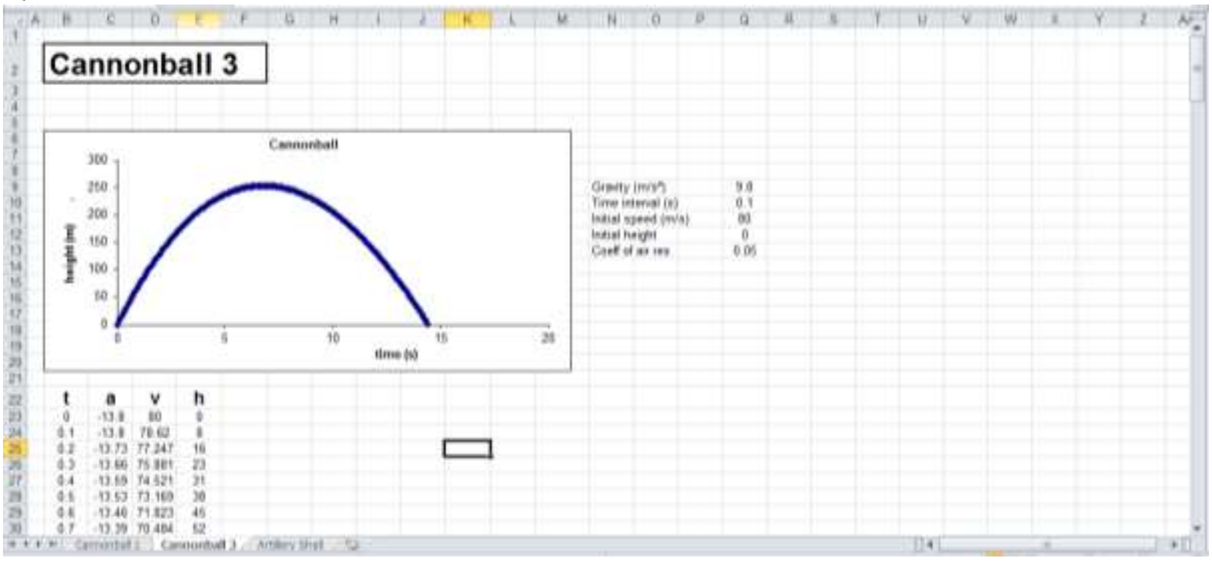
$$x_H(16.52) = 867m$$

Assumption is that the ground is level

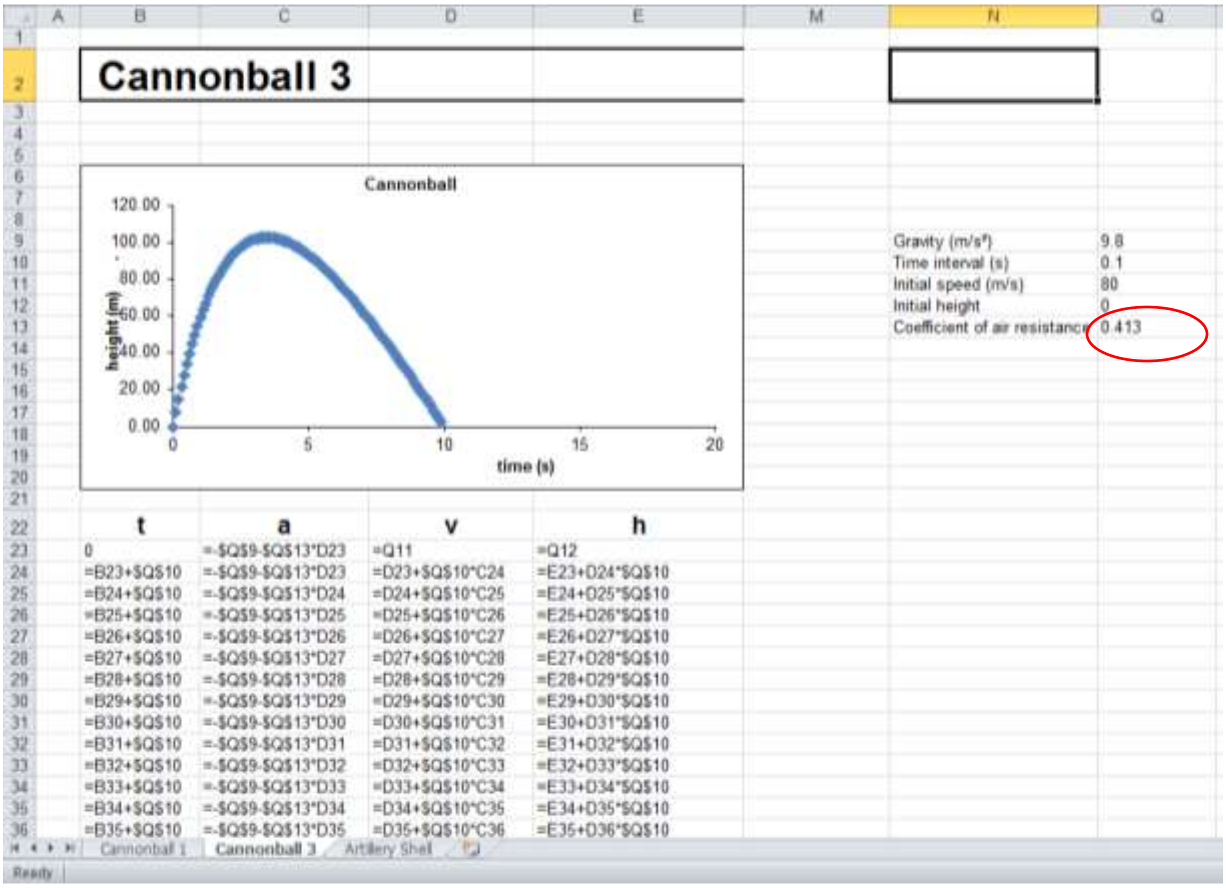
Q5.
The spread sheet 'Cannonball 1' does the following:
- calculates the velocity and height at regular time intervals
- graphs height against time
- Cells P9, 10, 11 and 12 represent gravitational acceleration, time interval, initial speed and initial height respectively (allows these variables to be easily changed)
- regular time intervals in column B starting at B23 and going up by P10
- velocity at time (respective B column value) starts at initial (P11) and each cell below is calculated by the previous velocity minus the change of velocity (P9*P10)
-height at time (respective B column) starts at initial (P12) and each cell below is the addition of the previous height and the change in height (velocity * time interval – respective C column*P10)

Strengths
- no calculus is required and therefore easily used
- changeable values dynamically
Limitations
- approximate result only (as the change in height uses instantaneous velocity at end of interval instead of average velocity over interval), can be improved with smaller time interval
Reasonable because the result between 16.2 and 16.3 is close to the calculus result of 16.33. If the model were refined by making the time intervals shorter, the result would be more accurate.

Q6.



Time taken to come back down: 14.42 s

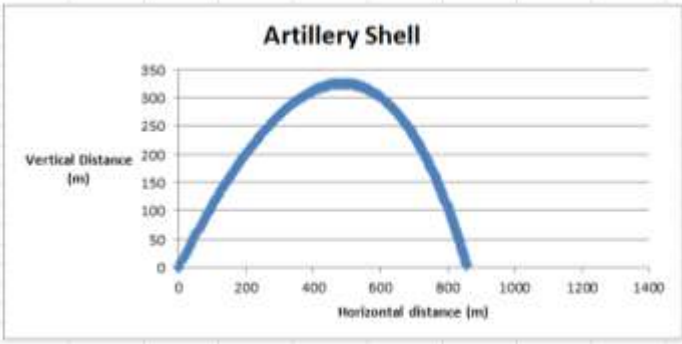


Required coefficient of air resistance: 0.413

The result of 14.42 s is quite reasonable when compared to the 14.57 from Q3.

Q7.

Artillery Shell



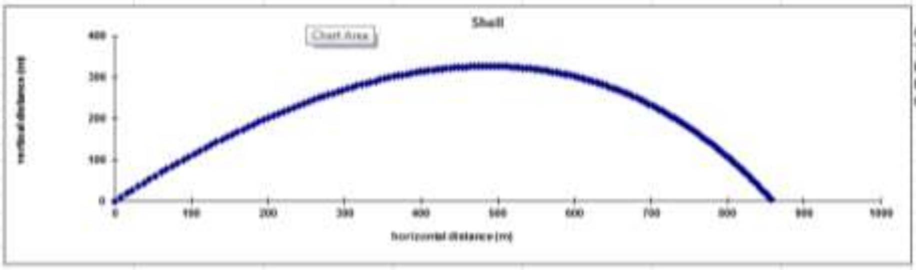
Gravity (m/s²)	9.8		
Time interval (s)	0.1		
Initial speed (m/s)	120		
Initial direction (deg)	50	(rad)	0.87
Coeff. of air res.	0.05		

t	x''	y''	x'	y'	x	y
0.00	-3.86	-14.40	77.13	91.93	0.00	0.00
0.10	-3.84	-14.32	76.75	90.49	7.67	9.05
0.20	-3.82	-14.25	76.37	89.05	15.31	17.96
0.30	-3.80	-14.18	75.98	87.63	22.91	26.72
0.40	-3.78	-14.11	75.60	86.21	30.47	35.34
0.50	-3.76	-14.04	75.23	84.80	37.99	43.82
0.60	-3.74	-13.97	74.85	83.39	45.48	52.16
0.70	-3.72	-13.90	74.47	82.00	52.93	60.36
0.80	-3.71	-13.83	74.10	80.61	60.34	68.42
0.90	-3.69	-13.76	73.73	79.22	67.71	76.34
1.00	-3.67	-13.69	73.36	77.85	75.04	84.12

Cannonball 1 | Cannonball 3 | Artillery Shell

Ready

Artillery



Gravity (m/s²)	9.8		
Time interval (s)	0.1		
Initial speed (m/s)	120		
Initial direction (deg)	50	(rad)	=M10/180*3.1415926
Coeff. of air res.	0.05		

t	x''	y''	x'	y'	x	y
0	=-$M$11*E24	=-$M$7-$M$11*F24	=M9*COS(O10)	=M9*SIN(O10)	0	0
=B24+$M$8	=-$M$11*E25	=-$M$7-$M$11*F25	=E24+$M$8*C24	=F24+$M$8*D24	=G24+$M$8*E25	=H24+$M$8*F25
=B25+$M$8	=-$M$11*E26	=-$M$7-$M$11*F26	=E25+$M$8*C25	=F25+$M$8*D25	=G25+$M$8*E26	=H25+$M$8*F26
=B26+$M$8	=-$M$11*E27	=-$M$7-$M$11*F27	=E26+$M$8*C26	=F26+$M$8*D26	=G26+$M$8*E27	=H26+$M$8*F27
=B27+$M$8	=-$M$11*E28	=-$M$7-$M$11*F28	=E27+$M$8*C27	=F27+$M$8*D27	=G27+$M$8*E28	=H27+$M$8*F28
=B28+$M$8	=-$M$11*E29	=-$M$7-$M$11*F29	=E28+$M$8*C28	=F28+$M$8*D28	=G28+$M$8*E29	=H28+$M$8*F29
=B29+$M$8	=-$M$11*E30	=-$M$7-$M$11*F30	=E29+$M$8*C29	=F29+$M$8*D29	=G29+$M$8*E30	=H29+$M$8*F30
=B30+$M$8	=-$M$11*E31	=-$M$7-$M$11*F31	=E30+$M$8*C30	=F30+$M$8*D30	=G30+$M$8*E31	=H30+$M$8*F31
=B31+$M$8	=-$M$11*E32	=-$M$7-$M$11*F32	=E31+$M$8*C31	=F31+$M$8*D31	=G31+$M$8*E32	=H31+$M$8*F32
=B32+$M$8	=-$M$11*E33	=-$M$7-$M$11*F33	=E32+$M$8*C32	=F32+$M$8*D32	=G32+$M$8*E33	=H32+$M$8*F33
=B33+$M$8	=-$M$11*E34	=-$M$7-$M$11*F34	=E33+$M$8*C33	=F33+$M$8*D33	=G33+$M$8*E34	=H33+$M$8*F34
=B34+$M$8	=-$M$11*E35	=-$M$7-$M$11*F35	=E34+$M$8*C34	=F34+$M$8*D34	=G34+$M$8*E35	=H34+$M$8*F35
=B35+$M$8	=-$M$11*E36	=-$M$7-$M$11*F36	=E35+$M$8*C35	=F35+$M$8*D35	=G35+$M$8*E36	=H35+$M$8*F36

Cannonball 1 | Cannonball 2 | Artillery Shell

Ready

The shell lands approximately 858m away

Q8.
Needs to be split into horizontal and vertical components as acceleration and velocity are different.

Vertical

$$\frac{dv}{dt} = -9.8 - 0.1v \quad where\ v\ is\ vertical\ velocity$$

$$\int \frac{dv}{9.8 + 0.1v} = -\int dt$$

$$10\ln(9.8 + 0.1v) = -t + c$$

$$e^{-\frac{t}{10}}e^{c} = 9.8 + 0.1v$$

$$at\ t = 0\ v = 50sin30$$

$$9.8 + 5sin30 = e^{c} \quad (= 12.3)$$

$$then\ v(t) = \frac{12.3e^{-\frac{t}{10}} - 9.8}{0.1}$$

$$v_V(t) = 123e^{-\frac{t}{10}} - 98$$

Hits ground when $x_V(t) = 0$

$$x_V = -1230e^{-\frac{t}{10}} - 98t + 1230$$

$$solve\ for\ x_V(t) = 0$$

$$t = 0\ (initial) \quad or\ t = 4.73s\ (next\ time)$$

Horizontal

$$\frac{du}{dt} = -0.1u \qquad where\ u\ is\ horizontal\ velocity$$

$$10\ln u = -t + c$$

$$u = Ae^{-\frac{t}{10}}$$

$$at\ t = 0\ u = 50cos30$$

$$A = 43.3$$

$$u = 43.3e^{-\frac{t}{10}}$$

$$x_H(t) = -433e^{-\frac{t}{10}} + 433$$

Horizontal distance travelled – sub in 4.73s into $x_H(t)$

$$x_H(4.73) = 163m$$

Just over 160m as per the question

Q9.

B25 = B24 + $K$6

C25 = -$K$9 * E24

D25 = -9.8 - $K$9 * F24   OR –$K$5 instead of 9.8

E25 = E24 + $K$6 * C24

F25 = F24 + $K$6 * D24

G25 = G24 + $K$6 * E25

H25 = H24 + $K$6 * F25