Algo for finding $f(x)$ of the form an^x+b

$2^x \times 3 + 5$

x	1	2	3	4	5	6	7			
y	11	17	29	53	101	197	389			
	3	6	12	24	48	96	192			

Multiplies by 2 $\therefore$ $2^x \times a + b$

$2 \times a + b = 11$

$4 \times a + b = 17$

$2a = 6$

$a = 3$

$\therefore b = 5$

Algo for finding $f(x)$ $f(x)$ is polynomial.

Difference method.

Algo for finding $f(x)$ $f(x)$ is periodic

$y = a \sin(\text{or } \cos) b(x+c)$

amplitude, phase shift, period $\rightarrow f(x)$

$y = a \tan b(x+c)$

a given by value at $\frac{1}{4}$ period

Finding Equations of Exponential and Polynomial Functions from Tables

Example

x	1	2	3	4	5	6	7
y	0	-3	-4	-3	0	5	12
	-3	-1	1	3	5	7	
	2	2	2	2	2		

Step 1 Write down the first differences.

If successive differences can be obtained by multiplying by the same number, say 5, then the function is exponential, of the general form, say $y = a \times 5^x + b$

If not, write down the second differences etc.
If you get to a line where all the numbers are the same, then the function is polynomial. The number of steps taken is the degree of the polynomial, ie the highest power of x.
Say the degree is 3, then the general form of the function is $y = ax^3 + bx^2 + dx + e$

Step 2 Decide how many unknowns you have and write that many equations by substituting pairs of values for x and y into the general form
Then solve the equations simultaneously to find a, b, c, etc

Shortcut Extend back to get the number pair for $x = 0$ and use that for one of the equations.

$y = x^5 + 2x^4 - 3x^2 + x - 5$

x	1	2	3	4	5	6
y	-4	49	376	1487	4300	10261

$y = ax^5 + bx^4 + cx^3 + dx^2 + ex + f$

$1^0 f + 1^1 e + 1^2 d + 1^3 c + 1^4 b + 1^5 a = -4$

$2^0 f + 2^1 e + 2^2 d + 2^3 c + 2^4 b + 2^5 a = 49$

etc

ie

$$\begin{pmatrix} 1^0 & 1^1 & 1^2 & 1^3 & 1^4 & 1^5 \\ 2^0 & 2^1 & 2^2 & 2^3 & 2^4 & 2^5 \\ 3^0 & 3^1 & 3^2 & 3^3 & 3^4 & 3^5 \\ 4^0 & 4^1 & 4^2 & 4^3 & 4^4 & 4^5 \\ 5^0 & 5^1 & 5^2 & 5^3 & 5^4 & 5^5 \\ 6^0 & 6^1 & 6^2 & 6^3 & 6^4 & 6^5 \end{pmatrix} \begin{pmatrix} f \\ e \\ d \\ c \\ b \\ a \end{pmatrix} = \begin{pmatrix} -4 \\ 49 \\ 376 \\ 1487 \\ 4300 \\ 10261 \end{pmatrix}$$

$$\begin{pmatrix} f \\ e \\ d \\ c \\ b \\ a \end{pmatrix} = \begin{pmatrix} a_{ij} = i^{j-1} \end{pmatrix}^{-1} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y \end{pmatrix}$$