

Maths C Diagnostic Test

- Q1. What is a group?
- Q2. Which group properties are and aren't satisfied by the set of even counting numbers under the operation of addition?
- Q3. Draw a Cayley table for the operation of addition modulo 5 on the residues modulo 5. Determine whether this system forms a group.
- Q4. The operation $*$ is defined on the real numbers by $a*b = a + b - 4$. Find the identity and the inverse of an element c .
- Q5. The operation \boxtimes is defined on the real numbers by $a\boxtimes b = \frac{a+b}{2}$. Find the identity or show that there isn't one.
- Q6. The set $\{a, b, c, d\}$ forms an Abelian group with a as the identity under the operation \diamond . Draw the Cayley table given that $d\diamond d=c$. Then redraw it given that $d\diamond d=a$ instead.
- Q7. An economy consists of two industries, steel and coal. It takes 0.8t of coal and 0.1t of steel to make a tonne of steel. It takes 0.3t of steel and 0.4t of coal to make a tonne of coal. The final demand is 40 000t of coal and 25 000t of steel. Use matrix methods to find the total demand for each.
- Q8. Without a calculator, express $\begin{vmatrix} 4 & i & 0 & 0 \\ -2 & 2a & 1 & 0 \\ -i & -2i & a & 0 \\ 17 & b & \sqrt{2} & 1 \end{vmatrix}$ in Cartesian form. Also give the answer in polar form if $a = -1$ and $b = 3\pi$.
- Q9. If $A = \begin{pmatrix} 1 & 2 & a \\ -1 & 0 & 2 \\ -2 & 1 & 0 \end{pmatrix}$, find A^{-1} .
- Q10. Express $\sqrt[3]{27cis\ 3\pi/4}$ in Cartesian form.
- Q11. In an arithmetic sequence, $t_3 = 32$ and $t_6 = 108$, find the lowest possible value for n such that the sum of the first n terms is greater than 1 000 000.
- Q12. In an arithmetic sequence, $t_7 = 40$ and the sum of the first 10 terms is 75. Find the 50th term.
- Q13. A bank account pays 8% pa interest compounding yearly on 1 January. Ethel put \$1000 into the account on 1 January 1990, then another \$1000 on 1 January 1991 and so on every year. How much did she have in the account on 2 January 2013?
- Q14. In a geometric sequence $t_4 = x+13$, $t_5 = x+3$, $t_6 = x-3$. Find t_2 .
- Q15. Write the recurring decimal 0.01454545454545... as a common fraction in simplest form.

Q16. The sum of the cubes of the first n odd numbers is a quartic polynomial function of n . Use matrix methods to find this polynomial.

Q17. In a sequence $t_n = t_{n-2} - t_{n-1}$. $t_3 = 4$ and $t_8 = -22$. Find t_{13} .

SOME ANSWERS

1. A group is a set with a binary operation defined on it where the operation is closed and associative, where there is an identity and where every element has an inverse.
2. Closed, associative, no identity, no inverse
3. Elements 0,1,2,3,4; first row 0,1,2,3,4; second row 1,2,3,4,0, then cycling around.
It does form a group.
4. $u=4$, $c^{-1} = 8-c$
5. No identity
- 6.
7. 90 000 t steel, 186 667 t coal
8. $9 + 6i$
- 9.
10. $\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}i$
11. 283
12. $2915/3$
13. 66 764.76
14. $625/9$
15. $4/275$
16. $2x^4 - x^2$
17. 246