

Exponential and Logarithmic Functions – Revision Sheet

Question 1

Work out / simplify the following. Don't use a calculator.

- a) 4^{-2}
- b) $64^{3/4}$
- c) $(5^{1/2})^6$
- d) $(1/4)^0$
- e) $\frac{r^2 r^{-1}}{r^{-4}}$
- f) $\log 100$
- g) $\log_5 125$
- h) $\log_4 1/4$
- i) $\log_{10} 5 + \log_{10} 20$
- j) $\log_2 80 - \log_2 5$
- k) $\log_r r^2$
- l) $\log_5 5^c$
- m) $\log_3 9^c$
- n) $\log_{1/4} 1/2$
- o) $\frac{\log_6 0.25}{\log_6 2}$
- p) $\frac{\log_3 25}{\log_9 5}$

Question 2

What is the meaning of $\log_6 60$? What is its value?

Question 3

What is the relationship between the operations of exponentiation base 2 ($x \rightarrow 2^x$) and taking logs base 2 ($x \rightarrow \log_2 x$)? How can this be used in simplifying expressions? Give some examples.

Question 4

Without your calculator, sketch the graph of $y = 2^x$ for $-4 < x < 4$.

On the same axes and with the same domain, sketch $y = (1/2)^x$ and $y = 2^{-x}$.

What do you notice? Can you explain why this is the case?

Question 5

A bacteria colony doubles its mass every 2 hours. Write a formula for its mass after t hours if its mass at $t=0$ is 0.05 g. By what percentage does its mass increase in a 10 minute period?

Question 6

The value of a car depreciates by 15% each year. If it is worth \$9 000 when it is 6 years old, how much was it worth new? How much will it be worth when it is 10 years old? When will its value drop below \$2000?

Question 7

Each step of 1 unit on the Richter scale corresponds to a $10 \times$ increase in the energy released in an earthquake. How many times more energetic is a magnitude 8.4 earthquake than a magnitude 6 earthquake?

Question 8

96% of a radioactive caesium sample decays in 90 hours. What is the probability that a given caesium nucleus will decay in any given hour?

Question 9

How much would I need to put into an account now to have \$6000 in 5 years if the account pays 4% interest compounding quarterly?

Question 10

I need to pay \$10 000 on 4 May 2009 and \$12 000 on 4 May 2012. How much must I put into an account on 4 May 2006 to have enough for these payments if the account pays 5.5% p.a. compounding yearly?

Question 11

I put \$1000 into a high risk investment that pays 100% per annum interest. Assuming nothing goes wrong, how much will I have at the end of the year if the interest compounds

- a) annually
- b) monthly
- c) weekly
- d) daily
- e) hourly
- f) every second

What is the growth factor over a year in case (f)?

Use the e^x button on your calculator to find e^1 (press SHIFT e^x 1 ENTER).

What do you notice?

Question 12

A weight is hanging on the end of a spring and is oscillating vertically such that the relation between height and time is sinusoidal. Its lowest height is 80 cm, its greatest height is 160 cm, and it takes 3 seconds to go up and 3 seconds to go back down, write the equation for the relation between height and time.

How would this equation be modified if the amplitude of the oscillation decayed exponentially (as weights on springs generally do) such that after 20 seconds the amplitude is halved?