

For each table below:

- a. find the differences
- b. find the rule.

Input	Output
0	1
1	4
2	7
3	10
4	13

Input	Output
0	3
1	8
2	13
3	18
4	23

Input	Output
0	5
1	12
2	19
3	26
4	33

These are harder.

The inputs start with 1

Input	Output
1	2
2	5
3	8
4	11
5	14

The inputs go up by 2

Input	Output
0	1
2	5
4	9
6	13
8	17

Challenge!

Input	Output
0	1
1	3
2	9
3	19
4	33

For each table below:

- a. find the differences
- b. find the rule.

Input	Output
0	1
1	4
2	7
3	10
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Input	Output
0	3
1	8
2	13
3	18
4	23

Input	Output
0	5
1	12
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The inputs start with 1

Input	Output
1	2
2	5
3	8
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The inputs go up by 2

Input	Output
0	1
2	5
4	9
6	13
8	17

Challenge!

Input	Output
0	1
1	3
2	9
3	19
4	33

## Calculus of Finite Differences

### Rules involving squared terms

1. For each table below:
  - a. find the 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> differences
  - b. find the rule.

Input	Output
1	4
2	7
3	12
4	19
5	28

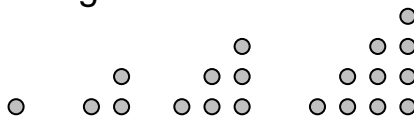
Input	Output
0	0
1	2
2	8
3	18
4	32

Input	Output
3	5
4	12
5	21
6	32
7	45

Input	Output
0	1
1	4
2	13
3	28
4	49

## 2. Triangular Numbers

The 1<sup>st</sup> triangular number is 1, the 2<sup>nd</sup> is 3, the 3<sup>rd</sup> is 6, the 4<sup>th</sup> is 10. The diagram shows why these numbers are called triangular numbers.:



On the right is a table showing the term number and the triangular number.

Term	Triangular Number
1	1
2	3
3	6
4	10
5	
6	

- a. Copy the table.
- b. Find the first, second and third differences.
- c. Now complete the table.
- d\*. Can you find the rule?
- e\*. What is the 100<sup>th</sup> triangular number?

## Calculus of Finite Differences – Rules of different types

1. For each table below:
  - a. find the differences.
  - b. find the rule.

Input	Output
1	2
2	5
3	8
4	11
5	14

Input	Output
0	7
1	8
2	11
3	16
4	23

Input	Output
2	13
4	25
6	37
8	49
10	61

Input	Output
1	1
2	16
3	41
4	76
5	121

### 2. Challenge – A salesman problem

John sells paintings every Saturday at a stall at a local market. Each Saturday he earns \$50, plus an extra \$20 for each painting he sells. Here is a table that shows how much he earns, based on how many paintings he sells:

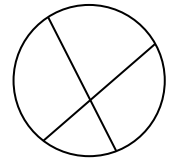
Number of paintings sold N	Total earnings E
0	50
1	70
2	90
3	
4	
5	

- a. Copy and complete the table.
- b. Find the differences
- c. Find the rule.
- d. Do you see a connection between the table and the rule? Explain.

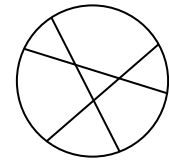
### 3. Super Challenge – Cutting a Pizza

Keegan has a big round pizza to share with some friends. He makes one cut, and cuts it into two pieces.

He then makes a second cut and sees that he now has four pieces. Not all of the pieces are the same size, but Keegan doesn't care. It is his pizza, so he will choose the biggest piece for himself.



Before making the third cut, Keegan realises that he can get seven pieces if the third cut doesn't pass through the point where the first two cuts cross. This should be clear from the diagram.



a. Here is a table containing the above information:

Number of cuts N	Maximum number of pieces P
1	2
2	4
3	7
4	
5	
6	

By drawing diagrams, find the maximum number of pieces for 4, 5 and 6 cuts. Enter the answers into the table.

b. What is the rule for finding the maximum number of pieces, P, if you know the number of cuts, N?

## Calculus of Finite Differences

The calculus of finite differences helps us to discover more complicated rules.

1. Complete each table. Calculate the first and second differences.

Rule:  $T = S \times 2 + 1$

Student	Teacher	First Differences	Second Differences
0			
1		_____	_____
2		_____	_____
3		_____	_____
4		_____	_____

Rule:  $T = S \times 3 + 2$

Student	Teacher	First Differences	Second Differences
0			
1		_____	_____
2		_____	_____
3		_____	_____
4		_____	_____

Rule:  $T = S \times 5 + 8$

Student	Teacher	First Differences	Second Differences
0			
1		_____	_____
2		_____	_____
3		_____	_____
4		_____	_____

- a. What can you say about the second differences?
- b. What do the first differences tell you about the rule?
- c. What other useful patterns have you discovered?

2. Using what you learned in question 1, find the rule for each of these tables.

a.

Student	Teacher
0	1
1	4
2	7
3	10
4	13

b.

Student	Teacher
0	3
1	7
2	11
3	15
4	19

c\*.

Student	Teacher
1	1
2	5
3	9
4	13
5	17

d\*.

Student	Teacher
2	15
3	17
4	19
5	21
6	23

3. The rules above are called linear rules, because if you graph them you get a straight line. Linear rules contain a **multiplying term** and an **adding term**. More complicated rules have a **squaring term**.

Complete each table. Calculate the first, second and third differences.

Rule:  $T = S^2 + 1$

Student	Teacher	First Differences	Second Differences	Third Differences
0				
1		_____	_____	_____
2		_____	_____	_____
3		_____	_____	_____
4		_____	_____	_____
5		_____	_____	_____

Rule:  $T = S^2 \times 3 + 2$

Student	Teacher	First Differences	Second Differences	Third Differences
0				
1		_____	_____	_____
2		_____	_____	_____
3		_____	_____	_____
4		_____	_____	_____
5		_____	_____	_____

Rule:  $T = S^2 \times 5 + 3$

Student	Teacher	First Differences	Second Differences	Third Differences
0				
1		_____	_____	
2		_____	_____	_____
3		_____	_____	_____
4		_____	_____	_____
5		_____		

- What do you notice about the third differences?
- What do you notice about the second differences?
- Do the first differences tell you anything about the rule?
- What other useful patterns have you discovered?

4. Using what you learned in question 5, find the rule for each of these tables.

a.

Student	Teacher
0	3
1	4
2	7
3	12
4	19

b.

Student	Teacher
0	1
1	3
2	9
3	19
4	33

c.

Student	Teacher
0	5
1	6
2	9
3	14
4	21

d\*.

Student	Teacher
1	3
2	15
3	35
4	63
5	99

5. Here is a rule that contains a squaring term, a multiplying term and an adding term. Complete the table. Then list any patterns that you have found.

Rule:  $T = 2S^2 + 3S + 4$

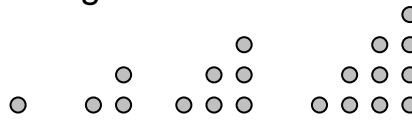
Student	Teacher	First Differences	Second Differences	Third Differences
0				
1		_____	_____	
2		_____	_____	_____
3		_____	_____	_____
4		_____	_____	_____
5		_____		



## Investigations – Some tricky rules!

### 1. Triangular Numbers

The 1<sup>st</sup> triangular number is 1, the 2<sup>nd</sup> is 3, the 3<sup>rd</sup> is 6, the 4<sup>th</sup> is 10. The diagram shows why these numbers are called triangular numbers.:



On the right is a table showing the term number and the triangular number.

Term	Triangular Number
1	1
2	3
3	6
4	10
5	
6	

- Copy and complete the table.
- Find the first, second and third differences.
- Can you find the rule?
- What is the 100<sup>th</sup> triangular number?