Algebra Problems

(1) If the equation $ax^2 + bx = c = 0$ has two real number solutions $x = x_1$ and $x = x_2$, write an equation (using a, b and c) whose solutions are $-x_1$ and $-x_2$. Explain your reasoning.

(2) An algebra class was asked to solve a quadratic equation of the form $x^2+ px + q = 0$. Tow students miscopied the problem - one miscopying only the coefficient of x, and the other miscopying only the constant term. The first found the answers to be 2 and 4. The second found the answers to be 2 and 7. If both solved their equations correctly, find the original equation and the correct roots.

(3) A student solved the equation (11-x)(x-4) = 6 as follows: 11-x = 6 or x-4 = 6. Hence x=5 or 6. The student then came up with another equation, e.g. (6-x)(x-9) = -4, which he solved in a similar way: 6-x = -4 or x-9 = -4 and hence x=10,5. In both cases, the answers are correct. Comment on the above solutions and find **all** the equations that can be solved in this way. (Explain your reasoning.)

(4) Here is an extended activity for class. The objectives include: practice in solving linear equations, generalizing, making conjectures and experiencing the element of surprise, combined with practice in executing the algorithm for solving two equations in two unknowns.

a. Divide your class into several groups, write the following systems of linear equations on the board and ask each group to solve one of the systems:

b. Ask each group to report its solution. Students will note that the systems have the same solution, x=-1, y=2

c. Now hand out the following worksheet:

(i) You have seen that each of the systems on the board has the same solution: x = -1, y=2. Based on these results write another system of equations whose solution is x= -1, y=2 and check that this really is the solution of your system.

(ii) Generalize the phenomenon in (i) and state it as a conjecture.

(iii) Justify or refute your conjecture in (ii).

d. After a while, have students compare their answers to (iii) and discuss the differences among the answers.

e. Discuss the following:

(iv) Does your answer in (ii) give all the systems whose solution is x = -1 and y =2 ? If so, show that this is the case. If not, find all the systems of linear equations whose solution is x = -1, y=2 and justify that you have all such systems. Write your answer using the concept of "if and only if" and in an equivalent form using the concept of "necessary and sufficient."

(v) Investigate what happens in case of a similar system of 3 equations in 3 unknowns.