Quadratic Expressions

Terminology
- sum, difference, product, quotient
- monomial, binomial, trinomial
- term and factor
- expression and equation

Expanding a binomial by a binomial
- Review of expanding
- Expanding a binomial by a binomial by a binomial.
- Expanding a trinomial by a trinomial
- Expanding perfect squares

Factoring quadratic expressions
- Review of factoring
- Quadratic expressions where a = 1
- Quadratic expressions where a <> 1
- Difference of squares

Calendar Magic

Given a row of even numbers, it always happens that the product of the first and third numbers is always four less than the square of the middle number. For example, 242 = 576, while 22 x 26 = 572, which is four less than 242.
Use algebra to show why this is always the case. Set your explanation out clearly.

If you look at a die, you'll notice that the numbers opposite to each other add to 7. That is, 1 is opposite 6, 2 is opposite 5, and 3 is opposite 4. This is the basis of this nice little trick.

Have a friend secretly roll two dice. Ask him to write down the answer to these four steps:
- multiply the two top numbers
- multiply the two bottom numbers
- multiply the top of one die by the bottom of the other
- multiply the other top and bottom

Now ask them to add up the four answers. The sum is always 49!
Use algebra to explain why the answer is always 49.
(Hints: use n and m to stand for the two top numbers. Then the two bottom numbers are 7-n and 7-m).

2. Here is a well known mental arithmetic trick.

	Example
To square a number consisting of a whole number and ½,	$(7\frac{1}{2})^2$
multiply the whole number by the next whole number, and add the fraction ¼.	= (7 x 8) + ¼ = 56¼.

Use algebra to show why it works.

4. **Multiplying Teen Numbers**

You can multiply any "teen" numbers (eleventeen to nineteen) in your head using the following pattern:

14	
x 17	mentally add the 1st number and the units digit of the 2nd number.
----	Tack on a zero. (14 + 7 = 21; tack on a zero gives 210)
210	
+ 28	add on the product of the units digits. (4 x 7 = 28)

238	the answer is the sum of 210 + 28 (238)

With a bit of practice you can multiply any teen numbers together in less than 5 seconds. Impress your friends, relatives and teachers!

a. Use algebra to show why it works.

b. Why it doesn't work for larger numbers?

5. Here's how to mentally square any number ending in 5.

Example 1	**Example 2**	**Procedure**
65^2	115^2	
--25	--25	The last 2 digits are always 25 (ie, 5^2).
6 x 7	11 x 12	Multiply the 10s digit by the next consecutive digit.
42--	132--	This gives the leading digits.
4225	13 225	And there's your answer!

Use algebra to show why this mental shortcut always works! (Hint: any number ending in 5 is of the form 10a + 5, where a stands for the number of 10s in the number. For example, 65 = 10 x 6 + 5, while 135 = 10 x 13 + 5.)

6. The above shortcut is in fact just a special case of a procedure for multiplying any numbers where all digits except the units digits are the equal, and the units digits add to 10.

Example 1	Example 2	Procedure
34 x 36	93 x 97	For Example 1, the tens digit is 3, and 4 + 6 = 10 For Example 2, the tens digit is 9, and 3 + 7 = 10.
4 x 6 = 24 --24	3 x 7 = 21 --21	Multiply the units digits together. This gives the last 2 digits of the answer.
3 x 4 = 12	9 x 10 = 90	Multiply the number of 10s by the next consecutive digit.
12--	90--	This gives the leading digits.
1224	9021	And there's your answer!

Use algebra to explain how this mental shortcut works!

Explanation of Teen Numbers

Teen numbers can be written in the form: 10 + a, where a stands for the units digit.

Therefore a normal teen multiplication is done like this: (note: a and b may be the same number)

```
       10 + a
    x  10 + b
   -------------
            ab
       10b
       10a
   100
  ------------------
100 + 10a + 10b + ab
```

We need to show that our mental shortcut gives us the same expression.

	(10 + a) + b	add the first number to the units
=	10 + a + b	digit of the second.
	10 (10 + a + b)	tacking on a zero is the same thing
=	100 + 10a + 10b	as multiplying by 10.
=	(100 + 10a + 10b) + ab	add on the product of the units
=	ab + 10a + 10b + 100	digits