

Factorising Trinomials

Because $2 \times 3 = 6$ we say that 2 and 3 are factors of 6.

Because $x(x + 2) = x^2 + 2x$ then x and $x + 2$ are the factors of $x^2 + 2x$

What are the factors of

$$x^2 + 3x$$

$$2x^2 + 5x$$

$$2x^2 + 6x$$

Note that the latter can be factorised in different ways in the same way that 12 can be factorised differently.

$$12 = 2 \times 6 \text{ or } 3 \times 4 \text{ or } 2 \times 2 \times 3$$

$$2x^2 + 6x = 2(x^2 + 3x) \text{ or } x(2x + 6) \text{ or } 2x(x + 3)$$

Show all the ways that the following can be factorised

$$4x^2 + 12x$$

$$20x - 5x^2$$

$$30x^2 + 20x$$

Because $(x + 2)(x - 2) = x^2 - 4$ then $x + 2$ and $x - 2$ are the factors of $x^2 - 4$.

What are the factors of

$$x^2 - 9$$

$$x^2 - 16$$

$$x^2 - 25$$

$$[x^2 - 11 \text{ if rational numbers are allowed}]$$

Does $x^2 + 4$ have any factors? (in the real number field). If so, what are they?

Because $(x + 4)(x + 3) = x^2 + 7x + 12$ then $x + 4$ and $x + 3$ are the factors of $x^2 + 7x + 12$

Etc etc

Note

If $x = 1$, then for the last example we get $5 \times 4 = 1 + 7 + 12 = 20$. That is it works/makes sense

If $x = 2$, we get $6 \times 5 = 4 + 14 + 12 = 30$

If $x = 3$, we get $7 \times 6 = 9 + 21 + 12 = 42$

That is the statement $(x + 4)(x + 3)$ is a set of different factors depending on the value of x .

Factorising Quadratic Expressions – Revision Questions

1. Common factors

Example: Factorise completely: $4x^2 - 2x$

Solution: Think: the common factor is $2x$.
 $4x^2 - 2x = 2x(2x - 1)$

2. Quadratic expressions of the form $ax^2 + bx + c$

Example: Factorise completely: $2x^2 - 7x - 15$

Any common factors? No!

For this expression, $a = 2$, $b = -7$ and $c = -15$

What multiplies to give “ac” and adds to give “b”?

$$\begin{aligned} &2x^2 - 7x - 15 \\ &= 2x^2 - 10x + 3x - 15 \\ &= 2x(x - 5) + 3(x - 5) \\ &= (x - 5)(2x + 3) \end{aligned}$$

| x ac | + b | |
|------|-----|------|
| -30 | -7 | |
| -30 | 1 | No! |
| -15 | 2 | No! |
| -10 | 3 | Yes! |

Factorising Quadratic Expressions

Name: _____

Time: 10 minutes

Reminder: Golden Rule of Factorising: Take out common factors first!

Common Factor: $ab \pm ac = a(b \pm c)$

Difference of squares $a^2 - b^2 = (a + b)(a - b)$

Perfect Square: $a^2 \pm 2ab + b^2 = (a \pm b)^2$

Factorise completely. Use the 'shortcuts' above to make it easier.

Q1) $4x^2 + 3x$

Q2) $x^2 - 9x + 18$

Q3) $2x^2 - 3x - 20$

Q4) $9x^2 - 16$

Q5) $x^2 + 10x + 25$

Q6) $3x^2 + 30x + 75$