Completing the Square

Solve $x^2 + 4x - 2 = 0$ by completing the square
Recall: $(x + a)^2 = x^2 + 2ax + a^2$. Note that the constant term is (half of the coefficient on x)^2.
We can use this pattern to solve quadratic equations.

$x^2 + 4x - 2 = 0$	We want to solve for x.
$x^2 + 4x = 2$	Move the constant to the RHS
$x^2 + 4x + 4 = 2 + 4$ (2 → 4)	Complete the square, adding the same quantity to both sides of the equality; halve the coefficient on the x-term , square it, and then add it to both sides.
$(x + 2)^2 = 6$	Write LHS as a perfect square.
$x + 2 = \pm\sqrt{6}$	Take the square root of both sides. Note this results in TWO equations.
$x = -2 \pm \sqrt{6}$	Make x the subject.

The two solutions are $x = -2 + \sqrt{6}$ and $x = -2 - \sqrt{6}$

Solve $2x^2 + 4x - 3 = 0$ by completing the square

$2x^2 + 4x - 3 = 0$	We want to solve for x.
$2(x^2 + 2x \quad) = 3$	Factor out the 2 on the LHS; move the constant to the RHS.
$2(x^2 + 2x + 1) = 3 + 2$ (1 → 1)	Complete the square, adding the same quantity to both sides of the equality; note we have added 2 to both sides.
$2(x + 1)^2 = 5$	Write LHS as a perfect square.
$(x + 1)^2 = \frac{5}{2}$	Divide both sides of the equation by 2.
$x + 1 = \pm\sqrt{\frac{5}{2}}$	Take the square root of both sides. Note this results in TWO equations.
$x = -1 \pm\sqrt{\frac{5}{2}}$	Make x the subject. Can you show that this equals $\frac{-2 \pm \sqrt{10}}{2}$?

Solve $3x^2 + 5x - 1 = 0$ by completing the square

$3x^2 + 5x - 1 = 0$	
$3(x^2 + \frac{5}{3}x \quad) = 1$	Factor out the coefficient on x^2; move constant to RHS.
$3(x^2 + \frac{5}{3}x + \frac{25}{36}) = 1 + \frac{25}{12}$ ($\frac{5}{6} \to \frac{25}{36}$)	Complete the square inside the brackets. We have added 3 x $^{25}/_{36}$ to both sides, which simplifies to $^{25}/_{12}$.
$3(x + \frac{5}{6})^2 = \frac{37}{12}$	Write LHS in factored form, as a perfect square; simplify RHS.
$(x + \frac{5}{6})^2 = \frac{37}{36}$	Divide both sides by 3.
$x + \frac{5}{6} = \pm\sqrt{\frac{37}{36}}$	Take square root of both sides.
$x = -\frac{5}{6} \pm \sqrt{\frac{37}{36}}$	Make x the subject of the formula.
$x = -\frac{5}{6} \pm \frac{\sqrt{37}}{6}$	Simplify the surd.
$x = \frac{-5 \pm \sqrt{37}}{6}$	Write as a single fraction.

Solve $ax^2 + bx + c = 0$ by completing the square
You'll get the quadratic formula!

First, completing the square is much more useful than the quadratic formula. Once you have y = a*(x+b)^2 +k you can easily answer all questions about the range of x values where y=c or y<c or y>c for many different c values, and you can can read off the max or min value of y for all x, and where it is attained, and you can see that y has the same value at both -b+h and -b-h (symmetry). The students I get often don't remember correctly the quadratic formula when they try to use it. I tell them to always use completing the square instead. You could also tell them that it reduces the complexity of the expression - before x appeared twice, after completing the square x appears only once in the quadratic expression. That is a tactic built into some CAS systems.

You might want to try letting "completing the square" fall out of something they may find simpler or more initially agreeable. For example. The easiest (and almost the only) useful factoring result (really multiplying result) is x^2 - y^2 = (x+y)*(x-y).
If you let S = x+y and T = x-y then we have [(S+T)/2]^2 -[(S-T)/2]^2 = S*T. (You can of course make that switch easier for students.)
What can you do with this? Let P = S*T in the above, so if P is an integer that is factored as S*T where S and T are integers also, then P is a difference of squares, but (S+T)/2 and (S-T)/2 will both be integers if and only if S and T have the same parity. With a little more thought, we see that an integer P is the difference of two square integers if and only if P is odd or an integral multiple of 4.
This leads to a factoring method for big integers.

Now suppose we have x^2+b*x =(x+b)*x then with S=x+b and T=x we see that x^2+b*x = [(b+2x)/2]^2 - [(b+x-x)/2]^2 =(x+b/2)^2 - (b/2)^2.
So we have completed the square. Of course you can now check directly that it works by expanding and collecting terms.