A Tricky Algebra Magic Trick

While my back is turned.

Write any number of 4 or more digits on the whiteboard.
Rearrange the digits anyway you please.
Subtract the smaller from the larger.

Now erase all of the numbers except the final answer.
Now erase any one digit of the final answer.

I turn around and reveal the missing digit!

Secret: add the remaining digits. the missing digit is the different between the total and the next highest multiple of 9.

Algebra:

For example

(1000a + 100b + 10c + d) – (1000c + 100a + 10d + b) = 900a + 99b – 990c – 9d = 9(100a + 11b – 99c – d)
Therefore the answer is divisible by 9, therefore the sum of the digits is a multiple of 9. If one is erased, I can restore it!

Why does this always result in a multiple of 9?

For each digit, we have 10^n * a – 10^m * a = (10^n – 10^m) * a = 10^m * [10^(n-m) – 1] * a. But 10^(n-m) - 1 is a multiple of 9, therefore the entire number is.

Why is 10^b – 1 a multiple of 9?
10^b – 1 = 10[10^(b-1)] - 1