

## Algebra Magic

File RA3-21

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1. Below is a list of even numbers.

... 12 14 16 18 20 22 24 26 28 30 32 34 36 38 ...

Circle any three consecutive numbers.

It happens that the product of the first and third numbers is *always* four less than the square of the middle number. For example,  $24^2 = 576$ , while  $22 \times 26 = 572$ , which is four less than  $24^2$ .

Use algebra to show why this pattern always holds, for *any* three consecutive even numbers.

2. Write down any four consecutive numbers (eg. 52, 53, 54 and 55). Multiply the 2 outer numbers together (eg.  $52 \times 55$ ). Multiply the two inner numbers together (eg.  $53 \times 54$ ). Repeat for other sets of 4 consecutive numbers.

- What pattern do you find?
- Use algebra to prove that this pattern will always be true.

3. If you look at a die, you'll notice that the numbers opposite to each other add to 7. That is, 1 is opposite 6, 2 is opposite 5, and 3 is opposite 4. This is the basis of this nice little trick.

Have a friend secretly roll two dice. Ask him to write down the answer to these four steps:

multiply the two top numbers  
multiply the two bottom numbers  
multiply the top of one die by the bottom of the other  
multiply the other top and bottom

Now ask them to add up the four answers. The sum is always 49!

**Use algebra to explain why the answer is always 49.**

## 4. Multiplying Teen Numbers

You can multiply any "teen" numbers (eleventeen to nineteen) in your head using the following pattern:

14	
x 17	mentally add the 1st number and the units digit of the 2nd number.
----	Tack on a zero. ( $14 + 7 = 21$ ; tack on a zero gives 210)
210	
+ 28	add on the product of the units digits. ( $4 \times 7 = 28$ )
-----	
238	the answer is the sum of $210 + 28$ (238)

With a bit of practice you can multiply any teen numbers together in less than 5 seconds. Impress your friends, relatives and teachers!

- Use algebra to show why it works.
- Why it doesn't work for larger numbers?

5. Here's how to mentally square any number ending in 5.

Example 1	Example 2	Procedure
$65^2$	$115^2$	
--25	--25	The last 2 digits are always 25 (ie, $5^2$ ).
$6 \times 7$	$11 \times 12$	Multiply the 10s digit by the next consecutive digit.
42--	132--	This gives the leading digits.
4225	13 225	And there's your answer!

Use algebra to show why this mental shortcut always works! (Hint: any number ending in 5 is of the form  $10a + 5$ , where  $a$  stands for the number of 10s in the number. For example,  $65 = 10 \times 6 + 5$ , while  $135 = 10 \times 13 + 5$ .)

6. The above shortcut is in fact just a special case of a procedure for multiplying any numbers where all digits except the units digits are the equal, and the units digits add to 10.

Example 1	Example 2	Procedure
$34 \times 36$	$93 \times 97$	For Example 1, the tens digit is 3, and $4 + 6 = 10$ For Example 2, the tens digit is 9, and $3 + 7 = 10$ .
$4 \times 6 = 24$ --24	$3 \times 7 = 21$ --21	Multiply the units digits together. This gives the last 2 digits of the answer.
$3 \times 4 = 12$	$9 \times 10 = 90$	Multiply the number of 10s by the next consecutive digit.
12--	90--	This gives the leading digits.
1224	9021	And there's your answer!

Use algebra to explain how this mental shortcut works!

## Explanation of Teen Numbers

Teen numbers can be written in the form:  $10 + a$ , where  $a$  stands for the units digit.

Therefore a normal teen multiplication is done like this: (note:  $a$  and  $b$  may be the same number)

$$\begin{array}{r} 10 + a \\ \times 10 + b \\ \hline \phantom{10} ab \\ 10b \\ 10a \\ 100 \\ \hline 100 + 10a + 10b + ab \end{array}$$

We need to show that our mental shortcut gives us the same expression.

$= (10 + a) + b$	add the first number to the units
$= 10 + a + b$	digit of the second.
$= 10 (10 + a + b)$	tacking on a zero is the same thing
$= 100 + 10a + 10b$	as multiplying by 10.
$= (100 + 10a + 10b) + ab$	add on the product of the units
$= 100 + 10a + 10b + ab$	digits