

Linear Models

1. The Rent-a-Wreck Auto Rental Company has the cheapest car rental rates in town.

You can choose one of two options.

- Option A - no flat fee, but a charge of 28c per kilometre.
- Option B - A flat rate of \$36 per day, plus 18c per kilometre.

a. Complete the following table, which compares the cost of hiring each vehicle for one day, for different distances driven.

	Distance Driven (hundreds of kilometres)								
	0	1	2	3	4	5	6	7	8
Option A Cost (\$)	0	28	56	84	112	140	168	196	224
Option B Cost (\$)	36	54	72	90	108	126	144	162	180

b. The *independent variable* goes on the X-axis, and the *dependent variable* goes on the Y-axis.

Does distance driven depend on cost, or does cost depend on distance driven?

.....

Which variable goes on the X-axis?

.....

c. On the grid paper:

- number and label the axes, and give the graph a title.
- plot the points for Option A, and join them with a straight line.
- plot the graph for Option B, and join them with a straight line.

d. Estimate from the graph - How far do you have to drive before Option B becomes cheaper?

- e. Find the gradient of the line for Option A. Show working Working

 Find the gradient of the line for Option B. Show working

 What does the gradient represent for this problem?

- f. What is the y-intercept of the graph of Option A?

 What is the y-intercept of the graph of Option B?

 What does the y-intercept represent?

- g. Find the linear equation for Option A
 Option B
- h. **CHALLENGE** - can you determine *exactly* how far you have to drive before Option B is cheaper? Show working on the right.

2. You have just written the world's greatest computer game. You must decide what price to put on it.

If you put on a high price you won't sell as many games, but you will make more profit per game sold. If you sell it for a low price you won't make as much profit on each sale, but you will sell many more copies of your game.

We say that 'The number sold depends on the price.' Economists often assume that the number sold and price form a *linear equation*.

From past experience, you think that if you sell it for \$160 per copy, you will sell about 800 copies. If you drop the price to \$40 per copy, you should sell about 8000 copies.

- a. On the graph paper, put Price on the X-axis, from 0 to 160. Use 0, 20, 40, ... 160. Put Number Sold on the Y-axis, from 0 to 8000. Use 0, 1000, 2000, ... , 8000

Why do we put No. Sold on the Y-axis?

.....

NB Don't forget to label your axes, and include a title for your graph.

- b. Plot the points A(160, 800) and B(40,8000). Since we assume this is a linear equation, join the points with a straight line.

- c. Use the graph to help you complete the table.

Cost	0	20	40	60	80	100	120	140	160
No. Sold			8000						800

- d. Revenue means total income. It is the product of Cost and No. Sold. Complete the table.

Cost	0	20	40	60	80	100	120	140	160
No. Sold			8000						800
Revenue									

- e. Estimate the revenue if you charged \$50 per copy. Estimating a value between two known values is called *interpolation*. Show your working.

.....

.....

.....

- f. Estimate the revenue if you raised the price to \$200 per copy. Estimating a value that is larger or smaller than known values is called *extrapolation*. Show your work.

.....

.....

.....

- g. Find the gradient and y-intercept of this graph.

- h. What is the physical interpretation of the gradient of this graph?

- i. Find the equation of this graph.

CHALLENGE

Not all graphs are straight lines.

- j. On this graph paper, plot the Price (X-axis) versus the Revenue (Y-axis).
- k. Join these points with a smooth *curve*.
 This curve is called a parabola.

Use this curve to estimate the maximum revenue.

- l. What price should you put on your software to get the maximum revenue? Show your working.

3. ANSWER THIS QUESTION ON YOUR OWN PAPER

A graph that converts from one quantity to another is called a conversion graph. Currently \$1 Australian buys \$.80 US.

- a. Make a table from \$0 Aus to \$1000 Aus, showing the \$US equivalent.
- b. Use the information in your table to draw the conversion graph.
- c. What is the gradient and the y-intercept of the graph?
- d. Find the equation that connects \$Aus and \$US.

Linear Models Assignment

Name: _____

Due Date: _____

Marking Criteria

Highly Satisfactory	has all of these:	<ul style="list-style-type: none">• neat• easy to understand	<ul style="list-style-type: none">• correct answer• explained well
Satisfactory	has most of these:	<ul style="list-style-type: none">• neat• easy to understand	<ul style="list-style-type: none">• correct answer• explained well
Unsatisfactory	has some of these:	<ul style="list-style-type: none">• neat• easy to understand	<ul style="list-style-type: none">• correct answer• explained well

Assignment Conditions

The assignment is to be done on lined A4 paper, and graph paper. You may use your notes, any books you wish, or a computer or calculator. You must not ask any other person for help, other than your teacher.

- A. Brock has \$_____ in the bank, is earning and saving nothing, and is spending about \$_____ a week on his new girl friend, Amber.
- B. His sister, Minerva, only has only \$_____ in the bank, but is spending nothing, and saves about \$_____ a week.

Do the following for both Brock and Minerva:

- Make a table showing how much they have after week 0, week 1, week 2, etc.
- Draw the graph showing how much each has in the bank after each week.
- For each graph, find the gradient of the line.
- What is the physical interpretation of the gradient?
- For each graph, find the y-intercept.
- What is the physical interpretation of the y-intercept?
- Find the equation of each graph.

Challenge Question

- h. When will Minerva have twice as much money as Bill?

Linear Models Solutions

1. The Rent-a-Wreck Auto Rental Company has the cheapest car rental rates in town.

You can choose one of two options.

- Option A - no flat fee, but a charge of 28c per kilometre.
- Option B - A flat rate of \$36 per day, plus 18c per kilometre.

a. Complete the following table, which compares the cost of hiring each vehicle for one day, for different distances driven.

	Distance Driven (hundreds of kilometres)								
	0	1	2	3	4	5	6	7	8
Option A Cost (\$)	0	28	56	84	112	140	168	196	224
Option B Cost (\$)	36	54	72	90	108	126	144	162	180

b. The *independent variable* goes on the X-axis, and the *dependent variable* goes on the Y-axis.

Does distance driven depend on cost, or does cost depend on distance driven?

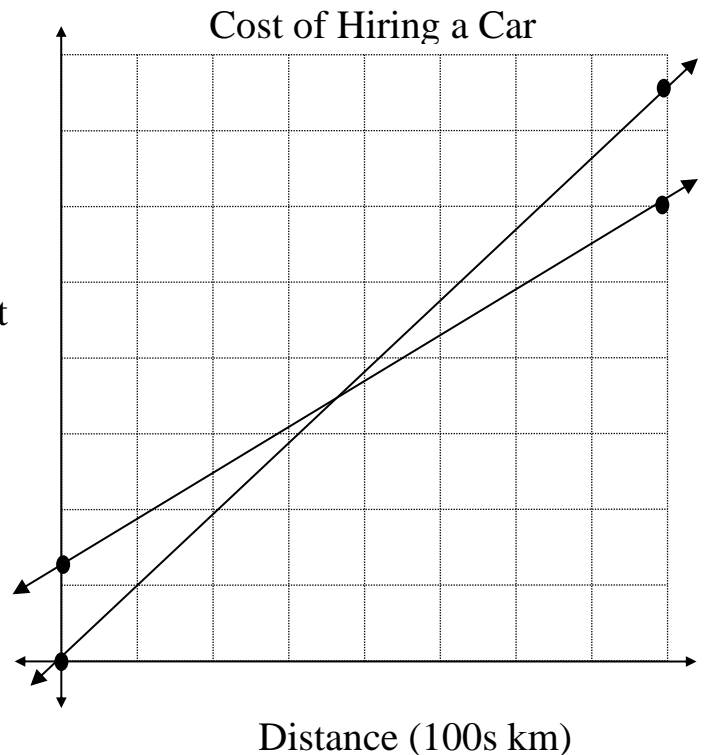
Cost depends on distance driven Cost

Which variable goes on the X-axis?

Distance

c. On the grid paper:

- number and label the axes, and give the graph a title.
- plot the points for Option A, and join them with a straight line.
- plot the graph for Option B, and join them with a straight line.



d. Estimate from the graph - How far do you have to drive before Option B becomes cheaper? **350 km**

- e. Find the gradient of the line for Option A. Show working **0.28**
- Find the gradient of the line for Option B. Show working **0.18**
- What does the gradient represent for this problem? **the cost per kilometre**
- f. What is the y-intercept of the graph of Option A? **0**
- What is the y-intercept of the graph of Option B? **36**
- What does the y-intercept represent? **The flat fee**
- g. Find the linear equation for Option A **C = 0.28 D**
- Option B **C = 0.18 D + 36**
- h. **CHALLENGE** - can you determine *exactly* how far you have to drive before Option B is cheaper? Show working on the right.

2. You have just written the world's greatest computer game. You must decide what price to put on it.

If you put on a high price you won't sell as many games, but you will make more profit per game sold. If you sell it for a low price you won't make as much profit on each sale, but you will sell many more copies of your game.

We say that 'The number sold depends on the price.' Economists often assume that the number sold and price form a *linear equation*.

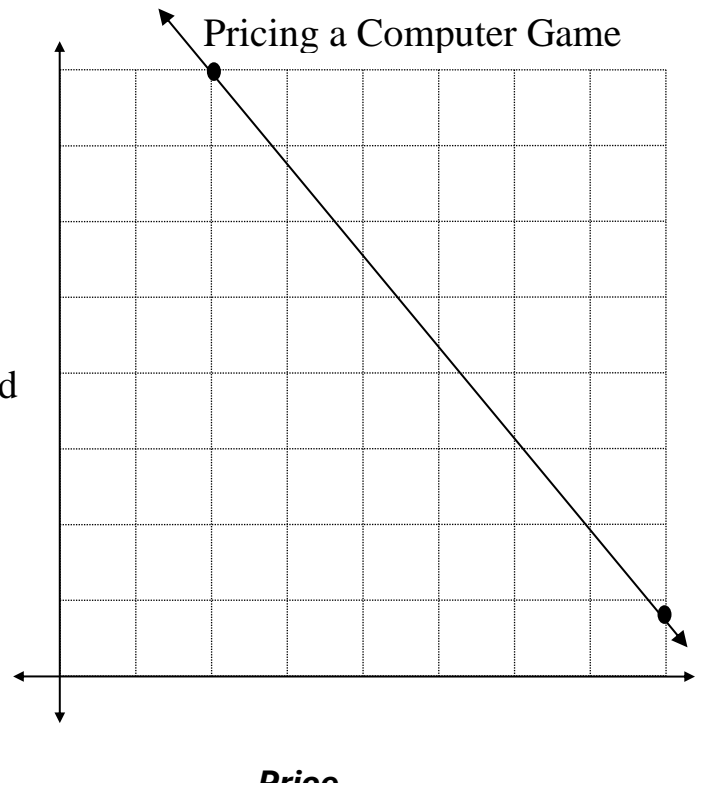
From past experience, you think that if you sell it for \$160 per copy, you will sell about 800 copies. If you drop the price to \$40 per copy, you should sell about 8000 copies.

- a. On the graph paper, put Price on the X-axis, from 0 to 160. Use 0, 20, 40, ... 160. Put Number Sold on the Y-axis, from 0 to 8000. Use 0, 1000, 2000, ... , 8000

Why do we put No. Sold on the Y-axis? **Because it is the dependent variable.**

NB Don't forget to label your axes, and include a title for your graph.

- b. Plot the points A(160, 800) and B(40,8000). Since we assume this is a linear equation, join the points with a straight line.



- c. Use the graph to help you complete the table.

Cost	0	20	40	60	80	100	120	140	160
No. Sold			8000						800

- d. Revenue means total income. It is the product of Cost and No. Sold. Complete the table.

Cost	0	20	40	60	80	100	120	140	160
No. Sold			8000						800
Revenue									

- e. Estimate the revenue if you charged \$50 per copy. Estimating a value between two known values is called *interpolation*. Show your working.

.....

- f. Estimate the revenue if you raised the price to \$200 per copy. Estimating a value that is larger or smaller than known values is called *extrapolation*. Show your work.

.....

