Introduction to Linear Equations

Teaching Sequence
It might be useful for students to do some parts of each session each day of the unit. This is because the problems in each session build up in complexity and student understanding is probably best developed over several days in each problem type.

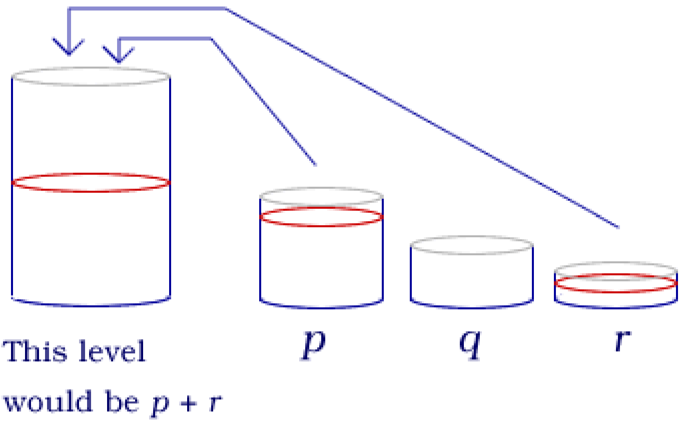
Session 1
Make up a set of three containers by trimming down 1.5 litre plastic soft drink bottles. It is preferable that these containers are of the same type. Straight-sided bottles are better than curved ones as they make it easier for the students to predict the relationships. The three cut down containers should hold different amounts of water when full. Make their capacity no more than 500 ml and ensure that these capacities are not multiples of each other, e.g. 120 ml, 230 ml, and 400 ml would be better than 100 ml, 200 ml, and 400 ml.

Label the containers p, q, and r respectively. Use them to set problems for the students by pouring from the p, q, and r containers into the large container.

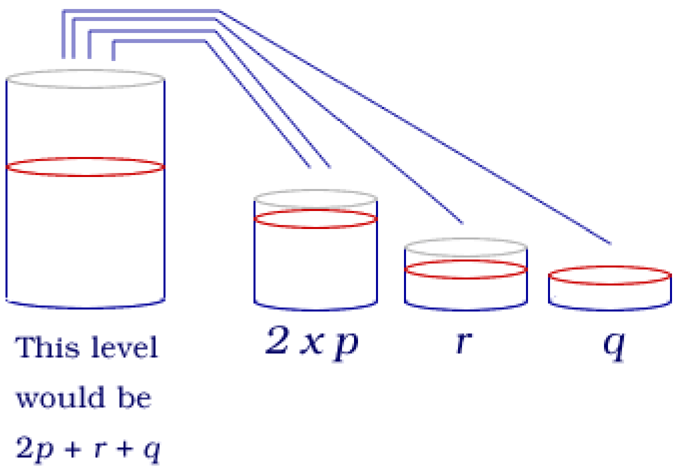
With each problem students must write an expression for the water level or do some other similar activity. Here are some examples. You can extend these as appropriate for your class.

First we give two examples.

A.



B.



1. Pour four lots of p into the 3 litre bottle and pour from it a container of r and a container of q. (This gives $4p – r – q$.)

2. Pour five lots of r into the large bottle and take out two lots of p. ($5r – 2p$)

3. Pour three lots of r and three lots of q into the large bottle and pour out two lots of p. (This gives $3r + 3q – 2p$ which could be written as $3(r + q) – 2p$.)

4. What does $3(r + q) – 2p$ mean in terms of pouring? (Put an r and a q lot together 3 times and then pour off 2 lots of p.)

   These types of problems allow opportunities for students to discover equivalence of expressions, particularly in the cases where order does or does not make a difference to the result, eg. $2r + p = p + 2r = r + p + r$.

5. Which of these are correct and which are false? Explain your answers.

   i. $3(r + q) – 2p$ is the same as $q + r + 2(r – p) + q$;
   ii. $4p – r – q$ is the same as $p + 2(p – r – q) + q + r + p$;
   iii. $2r + p + q = 2(p + q) + 3r – q – p – 2r$.

Session 2

Students look for patterns within each equation set and use these patterns to predict further equations in the set. They may do this using recursion, that is finding a relation between consecutive equations, rather than by looking for relationships within the equations themselves. Highlight relationships that might be found between the numbers in each set of equations and encourage the students to look for ways to describe these relationships. It is important that students find the unknowns using mental calculation, as this will help them to more easily recognise the relationships.

1. 1 – □= 1
   2 – □= 1
   3 – □= 1
   4 – □= 1
   …
   456 - □= 1

   Why is the right hand side of the equation always one?
   Use this pattern to solve:

   2000 - □= 1
   1001 - □= 2

2. 0 + 1 + 2 = □
   1 + 2 + 3 = □
   2 + 3 + 4 = □
   3 + 4 + 5 = □
   Is there anything in common between the number in the □'s?
   Why do you think this happens? (The number in the □is three times the middle number on the left of the equation.)
   a + b + c = 300 , a, b and c are different numbers.
   What numbers could they be?
   What values for a, b and c would fit the pattern?

3. 42 – 28 = □– 30
   52 – 28 = □– 30
   62 – 28 = □– 30
   ….
   92 – 28 = □– 30
   …
   712 – 28 = □– 30

What rule can you find for equations in this pattern.
How could this idea be used to solve 83 – 38?

4. 1 × 9 + 1 = □
2 × 9 + 2 = □
3 × 9 + 3 = □
4 × 9 + 4 = □
….
In □× 9 + □= 100, what is □if each □is the same number?
What rule can you find for all the equations in this pattern?

5. 1 = □÷ 11
2 = □÷ 11
3 = □÷ 11
4 = □÷ 11
…
In □= 77 ÷ 11, what is □?
In 33 = □÷ 11, what is □?

What rule can you find for equations in this pattern?
Would these equations be further down in the pattern? Give an explanation.□ = 111 ÷ 11

□= 111 ÷ 11
□= 1034 ÷ 11

Session 3

Students solve "What's my Number?" problems and record how they found the answer. At this stage trial and improvement are legitimate strategies though the problems are structured to encourage students to attend to structure and apply the processes of arithmetic.

1. If you take my number, multiply it by three then add seven, you get fifty-two.
What is my number? (15 since 15 x 3 + 7 = 52; or 3□ + 7 = 52, so 3□ = 45 and □= 15.)
2. If you take my number, subtract ten from it then divide it by two, you get sixteen.
What is my number? (42 since 42 – 10 = 32 and 32 ÷ 2 = 16)
3. If you take my number and add twenty-four to it, the answer is three times my number.
What is my number? (□ + 24 = 3□, so 24 = 2□ and □= 12.)
4. You take my number and divide it by three, the answer is the same as my number minus forty.
What is my number? (60 since 60 ÷ 3 = 20 and 60 – 40 = 20.)
5. If you take my number and multiply it by itself, the answer is my number added to itself.
What is my number? (2 since 2 x 2 = 4 and 2 + 2 = 4 and 0 for the same reason; or □x □= □+ □so □x □= 2□and □= 0 or □= 2.)

Session 4

This session involves students trying to work out the functional rule for given input and output numbers. These functions increase in complexity as the week progresses. Each example offers the students three input/output pairs. Below these three pairs are three other examples (highlighted in green) that could be used if needed. On the card that is presented to students staple a small piece of card over the output of these last three pairs so that students can predict the output number then check by turning up the cover.

1.

In	5	2	7	4	1	3
Out	9	3	13	7	1	5

The rule is two times the input number less one gives the output number

2.

In	2	5	9	4	1	3

Out	7	16	28	13	4	10

The rule is times three plus one.

3.

In	8	3	6	4	1	5
Out	5	0	3	1	-2	2

The rule is take away three.

4.

In	10	2	7	4	1	8
Out	6	2	4.5	3	1.5	5

The rule is divide by two plus one.

5.

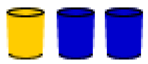
In	5	2	1	4	10	3
Out	25	4	1	16	100	9

The rule is the input number multiplied by itself (squared).

Session 5

These activities involve students in working out the number of counters or cubes that are in each cup of a given colour. Several clues are provided and students must combine these clues to find a solution. In each problem cover the top of the cups so students cannot look inside them. This is easily done by putting a screwed up piece of paper into the cup after the counters or cubes have been put in.
For each problem all students should solve it, recording their reasoning, before the solution is "revealed."

1. Into each yellow cup put four cubes, into each blue cup put five cubes.
   Clues:



14 cubes in total

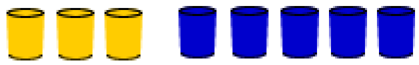
16 cubes in total

2. Into each yellow cup put three cubes, into each blue cup put six cubes. Clues:

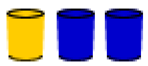
Four yellow cups have the same number of cubes as two blue cups.

15 cubes in total

3. Into each yellow cup put five cubes, into each blue cup put three cubes. Clues:



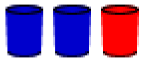
Three yellow cups have the same number of cubes as five blue cups.



11 cubes in total

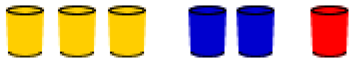
4. Into each yellow cup put two cubes, into each blue cup put four cubes, and into each red cup put six cubes.
   Clues:

Six yellow cups have the same number of cubes as three blue cubes that have the same number of cubes as two red cups.



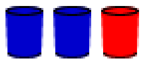
14 cubes in total

5. Into each yellow cup put three cubes, put one cube in each red cup and four in each blue cup. Clues:



Three yellow cups have the same number of cubes as one blue cup and two red cups

5 cubes in total



9 cubes in total

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Guess, Check, Refine

Working Backwards

Cover Up Method

Balance Method

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Linear Equations - Notes

I think that “many tools in your mathematical toolbox” needs to be highlighted here, as many methods are presented, and students need to learn to choose the most appropriate one.

Introduction to Equations – a joke

Here is a mathematical joke:

Q: How many repairmen does it take to change a light bulb?
A: Six - One to force it with a hammer and five to go out for more bulbs.

Q: How many mathematicians does it take to change a lightbulb?.
A: One. He gives the bulb to six repairmen, as this is a previously solved problem.

What, you didn’t find this funny?

What this joke tells you is how mathematicians think. If they have a problem, they try to use previously solved problems to help them answer it.

Guess – Check – Refine Method

What value of *y* makes this equation true: $\frac{3y-2}{5}=5$

Guess: $y = 1$

Check: $\frac{3\times1-2}{5}=\frac{3-2}{5}=\frac{1}{5}$ $y = 1$ is too small

Refine: Try $y = 10$

Check: $\frac{3\times10-2}{5}=\frac{30-2}{5}=\frac{28}{5}=5\frac{3}{5}$ $y = 10$ is just a bit too big

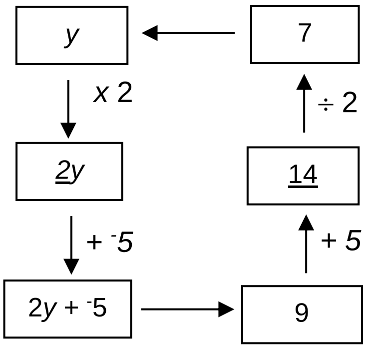
Refine: Try $y = 9$

Check: $\frac{3\times9-2}{5}=\frac{27-2}{5}=\frac{25}{5}=5$ $y = 9$ is the correct answer!

Backtracking method

Solve: 2y – 5 = 9

$2y - 5 = 9 \longleftrightarrow 2y + {}^{-}5 = 9$ Always change subtractions to additions first.



Always check your answer:

Check: 2 x 7 – 5 = 14 – 5

= 9

xxxx – should we insist throughout the text that we never have 2 equal signs on the same line?

Balance Method

see worksheet

There is only one rule - do the same thing to both sides of the equation.

Graphics Calculator Method

xxxx – this method will solve any equation with two variables. It often doesn't give the exact answer.

Cover-up Method

this method is particularly effective for questions like: $20 - 2y = 12$.

Learning process

Conjecture Investigate Confront Resolve
Multiple methods of doing a task

Linear Equations

In Year 8 – fill in the box, guess-check-refine, working backward, cover-up, doing the same op to both sides of equation. Variable on both sides.
In Year 9 – 3 step linear eqs; word problems, simple changing subject.
Year 10 – multiple steps, transposing equations, more complex changing subject.

Small Group Activity

You know how to solve questions of the form: $3y + 5 = 9$.
Explain or show how to solve these problems

$6 + 2y = 14$
$18 = 4y + 2$
$11 = 2 + 3y$

Exercises/Activities

We have $3 – 2y = 10$. John asks, "Can I just swap the 3 and the 2y?". Answer John's question.

Consider these two equations $2(y + 6) = 14$ $2(y+6) = 15$
John prefers to divide by 2 first.
Mary prefers to expand first.
Solve each of these questions both ways.
Choose the method you prefer and justify your choice.

Review- integer operations, adding and subtracting like terms, expanding brackets
Mathematical sentences – true, false, open
e.g. $y + 2 = 6$ is true if $y = 4$; is false if y <> 4
Equations, expressions, etc.
Equivalent equations
wrapping an equation - illustrate with a balance
Unwrapping an equation solving- order of ops in reverse
Balance- bags of lollies
Twiners Cookie jar
Lots of scaffolding to assist in setting out
One step equations,inc applications
two step equations,inc applications
Equations involving brackets and like terms
UnKnown on both sides of equation - balance method
Extension- e.g. $2-3y= 6$, inc cover-up method
Challenging word problems

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Year 9 Extension

Section on solving percent questions using linear equations with the formula

$Part = Percent \times Whole$ or $Part = Percent \times Original\ Amount$

and substituting and solving as necessary.

Martin Gardner – Killer Camels problem. Three posts at distances 10 m, 12 m and 14 m, with 3 gates.

Year 10 Extension

Look in the Technical Maths book for additional formulas for transposing

Leading problem - Scott’s Child Care Centre problem.