Magical Applications of Early Algebra

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One of the difficulties we face when teaching early algebra is the lack of applications. Too often when students ask, “Why do we need this?”, we have to say (or think we have to say), “Because you need to know this for next year.”.

My favourite applications of early algebra are magic tricks. Here are 6 magic tricks, all of which can be explained by the algebra of linear expressions.

1. Think of a Number Trick!

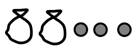
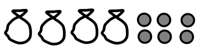
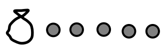
(adding and subtracting like terms; distributive law)

Think of a number.
Double it.
Add 3.
Double it.
Subtract 2.
Divide by 4.
Add 4.
Tell me your answer, and I will tell you your original number.

Secret: To find the original number, subtract 5 from the answer.

Use algebra to show why this trick works.

Solution

Step	**Example**	**Picture**	**Algebra**
Think of a number.	12		n
Double it.	24		$2n$
Add 3.	27		$2n + 3$
Double it.	54		$2(2n + 3) = 4n + 6$
Subtract 2.	52		$4n + 6 – 2 = 4n + 4$
Divide by 4.	13		$(4n + 4) \div 4 = n + 1$
Add 4.	17		$(n + 1) + 4 = n + 5$
(*Secret: Subtract 5*)	12		$(n + 5) – 5 = n$

Subtracting 5 at the end gives the original number!

2. Another Think of a Number Trick!

Use algebra to show why the following trick always works

Directions	Example	Picture	Algebra
1. Think of a number.	23		
2. Multiply it by 2.	23 *x* 2 = 46		
3. Add 2.	46 + 2 = 48		
4. Divide by 2.	48 ÷ 2 = 24		
5. Subtract 1.	24 – 1 = 23		
6. *The answer is the original number.*	It’s the same!		

3. Two Dice Magic! (adding and subtracting like terms; place value)

1. Roll a pair of dice.
2. Multiply the number on the top of one die by 2.
3. Add 3 to this result.
4. Multiply by 5.
5. To this result, add the number on the top of the second die.
6. Multiply this sum by 10.
7. Add 4.
8. What is your final answer?

Secret: *Subtract 150 from the final answer. The digits in the hundreds and tens places of the answer are the numbers rolled on the dice. The units digit is always 4.*

The Algebra

1.	Roll a pair of dice.	a and b
2.	Multiply the number on the top of one die by 2.	$2a$
3.	Add 3 to this result.	$2a + 3$
4.	Multiply by 5.	$5(2a + 3) = 10a + 15$
5.	Add the number on the top of the second die.	$(10a + 15) + \mathrm{b} = 10a + b + 15$
6.	Multiply by 10.	$10(10a + b + 15) = 100a + 10b + 150$
7.	Add 4	$10(10a + b + 15) = 100a + 10b + 154$
8.	Announce the result.	

Subtracting 150 gives the expression $100a + 10b + 4$. The first number is in the 100s place and the second number is in the 10s place. The units digit is always 4.

4. Subtraction Magic!

(adding and subtracting like terms, with brackets)

	Example
I will ask a student chooses any 3 digit number, where the first digit is different to the last digit. For example, let's say the number is 396.	396
Reverse the digits.	693
Subtract the smaller from the larger.	693 - 396 = 297
The answer is 297.	

I will do the calculation in my head while you do it on your calculator. Let's see who is fastest.

Sasha, please give us a 3 digit number where the 1st and last digits are different. (Sasha says 237). I immediately say the answer: 495.

Secret

The answer is always: 99 x (difference of first and last digits).

The Algebra

Students usually want to start by writing down *abc* as the 3 digit number in the first step. You will need some class discussion to get them to find the correct expression:

$$100a + 10b + c$$

Once over that hurdle, the rest of the solution is reasonably straightforward:

Original number	$100a + 10b + c$
Reverse the digits	$100c + 10b + a$
Subtract	$100a + 10b + c - (100c + 10b + a)$ $= 100a + 10b + c - 100c - 10b - a$ $= 99a - 99c$ $= 99(a - c)$

As an exercise, ask the students to do the proof if the 3rd digit is larger than the 1st.

5. A Magic Square! (adding and subtracting like terms)

At the start of this trick, I put a sealed envelope on the white board tray.

On an overhead transparency, I display the following grid:

3*a* - 1	**2*a* + *b* - 2**	**3*a* – 4**	**3*a* – *b* –1**
2*a* + *b*	***a* + 2*b* - 1**	**2*a* + *b* – 3**	**2*a***
2*a* + 2*b*	***a* + 3*b* -1**	**2*a* + 2*b* – 3**	**2*a* + *b***
***a* + *b* + 1**	**2*b***	***a* + *b* - 2**	***a* + 1**

I ask a student to choose any expression, e.g. $a + 2b – 1$. On the whiteboard, I circle that expression and put a big X through all the expressions in the same row and column.

I ask another student to choose any remaining expression, e.g. $3a – b – 1$. I circle that expression and put a big X through all the expressions in the same row and column.

I ask another student to choose any remaining expression, e.g. $2a + 2b$. I circle that expression and put a big X through all the expressions in the same row and column.

There is only one expression left, $a + b – 2$, so I circle it. I ask the students to add up the circled expressions. I write the answer on the whiteboard.

I now ask the student with the envelope to open it and write on the whiteboard what is on the piece of paper. It is the sum of the 4 circled expressions!

Secret

It is just an addition table! Before the lesson, I made up the following addition table. If you complete the table, you will get the table above. The sum of the circled expressions is the sum of the 8 original expressions. Can you see why?

+	***a***	***b* - 1**	***a* - 3**	***a* - *b***
2*a* – 1	~~3*a* - 1~~	~~2*a* + *b* - 2~~	~~3*a* – 4~~	(**3*a* – *b* –1**)
a* + *b	~~2*a* + *b*~~	(***a* + 2*b* - 1**)	~~2*a* + *b* – 3~~	~~2*a*~~
a* + 2*b	(**2*a* + 2*b***)	~~*a* + 3*b* -1~~	~~2*a* + 2*b* – 3~~	~~2*a* + *b*~~
***b* + 1**	~~*a* + *b* + 1~~	~~2*b*~~	(***a* + *b* - 2**)	~~*a* + 1~~

Once they understand how this Magic Square was constructed, challenge each student to make up their own Magic Square and find the Magic Sum.

6. Calendar Magic! (adding and subtracting like terms)

Consider the calendar alongside (any calendar will do).

July 2003						
S	**M**	**T**	**W**	**T**	**F**	**S**
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

a. Choose any 3 dates in a row (an example is shown). Add the two outer numbers. Double the middle number. What do you notice? Does this pattern always hold? Use algebra to prove it.

b. Chose any 3 dates in a column. Add the top and bottom numbers. Double the middle number. What do you notice? Does this pattern always hold? Use algebra to prove it

c. Choose any 5 dates in a row. Find a pattern that fits any 5 such dates. Use algebra to show why it always holds.

d. Here is a magic trick. Secretly choose any 2 x 2 square of numbers. Tell me the total. I can tell you the numbers you chose. How do I do it? Use algebra to discover how my trick works.

Secret *If n is the number in the upper left corner of the square, then the number to the right is n +1, the number below is n + 7 and the number in the lower right corner is n + 8. The sum of the 4 numbers is 4n + 16. To find the value of n, subtract 16 (subtract 20, add 4) and divide by 4 (divide by 2, divide by 2).*

7. More Calendar Magic

The first activity in this investigation is, algebraically-speaking, somewhat challenging. Its purpose is to 'hook' the students – they always want to know 'how did you do that?'

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Here is a magic trick. From the calendar alongside (or any calendar) secretly choose any 2 x 2 square of numbers. Tell me the total. I can instantly tell you the numbers you chose.

For example, if you tell me that your total is 52, I will tell you that your numbers are 9, 10, 16 and 17.

July 2003						
S	**M**	**T**	**W**	**T**	**F**	**S**
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

The Secret: *I divide your total by 4, and then subtract 4. This gives me the first number you chose (in the above example, 52 ÷ 4 = 13; 13 – 4 = 9). I add 1 to get the next number (9 + 1 = 10). I add 7 to the original number to get the 3rd number (9 + 7 = 16), and then I add one more to get the 4th number (16 + 1 = 17).*

The Algebra: *If n is the number in the upper left corner of the square, then the number to the right is n +1, the number below is n + 7 and the number in the lower right corner is n + 8. The sum of the 4 numbers is 4n + 16. To find the value of n, subtract 16 (subtract 20, add 4) and divide by 4 (divide by 2, divide by 2).*

Even easier, re-write 4n + 16 as 4(n + 4). To find the value of n, divide by 4 (divide 2, divide 2) and then subtract 4.

Consider the calendar alongside (any calendar will do).

Calendar Magic Handouts

Apr-06

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30						

May-06

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Jun-06

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

Jul-06

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

Apr-06

S	M	T	W	T	F	S
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2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
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May-06

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7	8	9	10	11	12	13
14	15	16	17	18	19	20
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28	29	30	31			

Jun-06

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4	5	6	7	8	9	10
11	12	13	14	15	16	17
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Jul-06

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