Fun with Calendars – An Algebra Puzzle

A fun mathematical puzzle to play with your friends. (Or teachers, with your class.)

Take any calendar. Tell your friend to choose 4 days that form a square like the four below. Your friend should tell you only the sum of the four days, and you can tell her what the four days are.

January 1998						
S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24

How does the puzzle work? You know how people always want to see a use for algebra? Well this puzzle uses algebra. Here's what I mean.

Let's pretend that the 4 numbers that the person chose were the highlighted ones here - 18, 19, 25, and 26. She adds up the four numbers and tells you only that the sum is 88.

You make a couple of calculations and tell her the numbers. What calculations? Lets figure that out with algebra. Let's call the first number n. Then you know that the next number would be n + 1 and the next number would be n + 7 and the next number would be n + 8. We had our friend add up the four numbers, so let's add our four numbers:

$n + n + 1 + n + 7 + n + 8$

And since our friend got 88 when she added them, let's make our sum equal 88:

$n + n + 1 + n + 7 + n + 8 = 88$

Simplify our equation by adding like terms:

$4n + 16 = 88$

How would you solve this equation? Subtract 16 from both sides?

$4n = 72$

Divide both sides by 4?

$n = 18$

Subtract 16 and divide by 4. That's exactly how you solve the puzzle. When your friend tells you the sum, you subtract 16 then divide by 4. This gives you the first number n. (Then add 1 and 7 and 8 for the other numbers).

Alternate and easier method. Subtracting 16 mentally isn't so easy. Go back to that equation:

$4n + 16 = 88$

I think I see a better way. Factor 4 from the left side of the equation:

$4(n + 4) = 88$

Now, I could divide both sides by 4:

(n + 4) = 22

Subtract 4 from both sides.

n = 18

That's a lot easier to do mentally. Divide by four and then subtract 4.

So how does the puzzle work again? Your friend adds any 4 numbers that form a square on the calendar and tells you the sum. You divide by four and then subtract 4. That gives you the first number. You add 1, 7, and 8 to get the other numbers.

And algebra makes it all possible.

**

Calendar Magic

Every 4th year is leap unless it's divisible by 100. This is only true with one additional caveat: among those divisible by 100, the ones that are divisible by 400 are still leap. As every one can see, this adds to the argument that calendars are deeply involved with mathematics. Or, perhaps, it's the other way round. Below there is a Java monthly calendar that may be used to discover some simple mathematics built into calendar tables. Due to a bug in (my) Java libraries, the calendar only works for the years between 1970 and 2037 (inclusive). It's more than enough to make a few math discoveries.

This is how you use the applet. When the second button on the left reads "Set", drag the cursor inside the table and select a square array of dates. Once you are satisfied with your selection, click the button. The label will change to "Play". While playing pick up dates inside the select area. One in each row and in each column. Sum up selected dates. The sum that also appears in the lower right corner does not depend on selection of dates inside the square but only on the square itself. See if you can verify or even prove this.

January 1998

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

**

A calendar table has other interesting properties. For example, for any three dates located successively in a row or a column or even diagonally, the middle one serves as an average of the other two.

Selecting dates so that there be only one per row and only one per column is equivalent to placing rooks on a chessboard so that they do not threaten each other. So the puzzle may be played on boards larger than 4x4 (maximum available with a calendar device) provided the fields are numbered in a proper way. How should the fields be numbered to preserve the properties we observed on the calendar boards?

For this one you may need a calculator. Select three consecutive dates in a row, column, or diagonal. Write them one beside another to form a single number. This is number One. Select another three dates and write them down to get number Two. Multiply number One by number Two (this is where you'll need a calculator). Give me the list of digits of the product skipping just one digit. I'll tell you which digit you have skipped.

Explanation

Everything is based on properties of arithmetic progressions(or series). These are sequences of numbers with a fixed difference between any two consecutive members. In a row the difference is 1, in a column 7, on diagonals 6 and 8 (or -6 and -8 depending whether you count downwards or upwards.) Three consecutive terms in any arithmetic series can be written as a, a+d, a+2d. Then the average (a + (a+2d))/2 of the outer two equals (a+d) which is the middle term. From here it also follows that the sum of three successive numbers is (3a+3d) and is divisible by 3.

Since the number is divisible by 3 iff the sum of its digits is divisible by 3, should we write three dates located successively in a row, or a column, or diagonally, the resulting number will be always divisible by three. The product of two such numbers will therefore be divisible by 9. Thus, the sum of its digits will be also divisible by 9. So it's easy to detect a missing digit.

In a, say, 4x4 square the dates can be written as

(a)	(a)+1	(a)+2	(a)+3
(a+7)	(a+7)+1	(a+7)+2	(a+7)+3
(a+14)	(a+14)+1	(a+14)+2	(a+14)+3
(a+21)	(a+21)+1	(a+21)+2	(a+21)+3

If we choose four dates following the rule that only one is selected per row and only one per column, when we add them up, there will be only one in the form (...)+1, only one in the form (...)+2, and so on. Inside parentheses we'll have only one (a) and only one (a+7) and so on. Therefore, the sum will always be equal to 4a + (0+7+14+21) + (0+1+2+3) = 4a + 48.

The problem of selection of elements 1 per row and 1 per column is the same as constructing a Latin square of a given size. The same idea works for a special case of the Toys and Tots problem (William A McWorter Jr.)

> n toys are to be distributed among k children. For i=1,...,n, the i-th toy is worth i dollars. To please the parents, the total dollar value of the toys each child receives should be exactly the same. To please the children, each child should receive no more than one more toy than any other child receives. Determine when such a distribution is possible and describe a distribution when it is possible.

In the case where n is divisible by k2, there is a very elegant solution. Arrange the n toys in a k by n array where the dollar value of each toy increases from left to right and top to bottom. Here is a sample arrangement for k=3 and n=27.

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27

Now, using the children's names as symbols, form any k by k latin square. For example, for k=3 and names Dawn, Dee, and Mary,

Dawn	Dee	Mary
Dee	Mary	Dawn
Mary	Dawn	Dee

Cover the array with three copies of this latin square and give each child those toys under his or her name. By calendar magic, the 9 toys each child receives have the same total dollar value. For the example above, the table below gives the toys each child receives, the total value each child receives being $126.