

Early Algebra

Preliminary

What is arithmetic?

What is algebra? algebra is a way of showing patterns in arithmetic

Student and Teacher

first with words

gradually introduce variables

introduce implied multiplication and fraction bar as division

different rules that give the same answer - discuss

advantage of giving numbers in order, e.g. making a table.

scatterplots and graphs of rules.

constant function

introduce powers

Exploring Number Patterns

don't use implied multiplication yet!

trestle tables

toothpicks

Bob Dengate's example - email AAMT.

revise: repeated addition --> multiplication and repeated subtraction --> division

English expressions to algebra; e.g. three times a number plus one, four times a number minus three.

XXXX - needs more work

Function Machines

find output for different inputs, including fractions and integers (evaluate)

tables, functions, graphs

guess the rule

given output, find input (solve)

regression - given table, find equation.

applications - direct proportion, fixed and variable costs, supply and demand, perimeter formula.

evaluating functions exercises.

Expanding - Distributive Law

often same thing can be expressed in more than one way

introduce bags and stones

$*$ = unknown number of stones in a bag. $* * *$ represents number of stones in 3 bags, or $3 \times *$ or $3*$.

$* + 5$ represents a bag with an unknown number of stones + 5 more stones.

$* - 2$ represents a bag with an unknown number of stones initially, with 2 stones removed.

use bags and stones to show that $2(* + 3) = 2* + 6$

use bags and stones to represent $2(* - 3) = 2* - 6$.

use patterns, tables and graphs to confirm that $2(* - 3) = 2(* + -3) = 2* + -6 = 2* - 6$. confirm with table and graph.

Simplifying expressions - Adding and subtracting like terms

$3 + 4 + 5 = 3 + (4 + 5) = 3 + 5 + 4 = 3 + (5 + 4) = 4 + 3 + 5 = 4 + 5 + 3 = 5 + 3 + 4 = 5 + 4 + 3$.
you can add in any order.

example: $14 + 89 + 86 = (14 + 86) + 89 = 189$; $13 + 39 + 57 = (13 + 57) + 39$. Commutative law of addition.

XXXX However $(10 - 6) - 3$ and $10 - (6 - 3)$. You must subtract in the order specified.

revise $3y = 3 \times y = y + y + y$

$3y + 4y = y + y + y + y + y + y + y = 7y$

$3 \times 2 \times 4 + 4 \times 2 \times 4 = 7 \times 2 \times 4 \rightarrow 3xy + 4xy = 7xy$

$3 \times x^2 + 4 \times x^2 = 7 \times x^2$

which all lead to the rule for adding and subtracting like terms..

$3 \times a + 4 \times b = a + a + a + b + b + b + b = 3a + 3b$, etc.

which leads to the rule: you can't add or subtract unlike terms.

word problems: 3 times some number +4 times some number; $b =$ no of boys, $g =$ no of girls,
expression for number of students.

Simplifying expressions - multiplying and dividing

$2 \times 3 \times 4 = 2 \times 4 \times 3 = 4 \times 2 \times 3$; $abc = acb = bca = cba = bac = cab$.

example: $4 \times 17 \times 25 = (4 \times 25) \times 17 = 1700$; $2 \times 45 \times 50 = (2 \times 50) \times 45 = 4500$. Commutative law of multiplication.

$(2 \times 3) \times 4 = 2 \times (3 \times 4) = 24$.

$(a \times b) \times c = a \times (b \times c) = abc$

multiplying with powers - $a^2b \times a^3b^3$

dividing algebraic expressions - e.g. $6/3 = 2 \times 3/3 = 2$, $ab/b = a$, $2(a+b)/(a+b) = 2$, $2(a+b)/2 = a+b$.

Notes programming a GC is a great way to reinforce the concept of a variable.

Assignment - Crossing the River