M1 Maths

Skills – Communicating

- communicate mathematical ideas effectively and in conventional style
- prove mathematical statements

Communicating **Proof**

Communicating

When answering a question or solving a problem that requires mathematical reasoning, sometimes just getting the answer is all that is needed. At other times, though, we need to communicate our reasoning as well so that someone else can see how we got our answer and so that they can see that our reasoning is sound and therefore our answer is correct.

This means ***showing our working***, so that others can follow our reasoning. Sometimes this is called ***justifying*** our answer. Showing working or justifying means writing a series of mathematical statements, the first being something that we are told (given information), each subsequent statement following in an obvious way from earlier ones, and the final statement being our answer to the question.

This sequence of statements often needs to be interspersed with sentences in English making it clear what we are doing, so that others can follow our working without having to stop and work out what's going on.

We need to follow conventions of spelling, grammar etc., just like we would if writing an essay in an English lesson. We also need to follow certain conventions specific to mathematical communication. Examples of such mathematical conventions are:

- using the proper abbreviations for units, e.g. km, m^2, mL, ha etc.
- writing '=' signs underneath one another when doing a long calculation
- making sure that every statement or line of working is true by itself.

To illustrate the last point, it is a common mistake when calculating say $12 \times 3 + 17$ to write:

$$12 \times 3 = 36 + 17 = 53$$

This says that $12 \times 3 = 36 + 17$, which, of course, is not true.

A better way to write this is

$$12 \times 3 = 36$$
$$36 + 17 = 53$$

Having just one '=' sign per line is a good guide and will avoid this type of mistake, though there are situations where this is not essential. Writing

$$12 \times 3 + 17 = 36 + 17 = 53$$

is acceptable and does save space.

Remember that communication is a significant part of your assessment in maths. Once you've got the hang of good communication, to some extent, you tend to do it without thinking about it. This can provide you with easy marks on many tests at all levels.

Below is a list of dos and don'ts which relate to common errors in mathematical communication.

Some Communication Dos and Don'ts

Use an appropriate method	
Do	**Don't**
If a question tells you to use a particular method, use it and make it clear that you did so	If a question tells you to use a particular method, you won't get credit for using a different method.
If a question doesn't specify a method, you may use any valid method including guess and check or any calculator function, but make it clear what method you did use. Words will help make it clear	
Show working that any person with reasonable mathematical knowledge can follow easily	Just give the answer.
Show what steps are involved and give some details of which bits you used the calculator for and how	Just say 'Done with a calculator' in place of several steps

Explain in words what you are doing	
Do	**Don't**
Write as if you were explaining to someone who is not as clever as you how to do the question	Just write line after line of symbols

Write legibly in pen or dark pencil and large enough for old people to read	
Do	**Don't**
Use a 2B pencil	Use a 2H pencil
Use blue or black pen	Use red pen
Use two lines for rational expressions (fractions)	Cram fractions into one line
Make your work as easy to read as possible.	

Lay your work out tidily	
Do	**Don't**
Use the lines	Write across lines
Think where would be a good place to put diagrams	Put diagrams just anywhere

Put all successive line of working underneath each other	Put lines of working beside other lines of working as it is not always clear which should be read first
When solving equations or manipulating expression over a number of lines, put the = signs underneath each other	

Use proper English (especially mathematical English)

Do	Don't
Use the same care with grammar and spelling that you would in an English lesson	Write ungrammatical English
you subtract 3 add 4 to x multiply it by 5 differentiate x^2 to get $2x$	Use words incorrectly, e.g. then you minus 3 plus 4 to x times it by 5 derive x^2 to get $2x$
then multiply by 5	Use symbols as shorthand for words in word sentences e.g. then × 5

Use the symbol '=' correctly

Do	Don't
Use = only between two symbolic expressions to indicate that they have the same value (come to the same number when worked out) e.g. $x + 4 = 9$. Otherwise use words e.g. let the width be w *t* is the time in seconds since the ball was thrown	Use it in a word sentence to mean *is*, *are*, *be* etc. e.g. let the width = *w* *t* = time
$5 \times 6 = 30$ $30 + 2 = 32$ $y = x^2$ $\frac{dy}{dx} = 2x$	State that two expressions are equal when they are not, e.g. $5 \times 6 = 30 + 2 = 32$ $y = x^2$ $= 2x$

Define abbreviations

Do	Don't
Define all abbreviated quantity names not used in the question. Either define them in a statement or show on a diagram what they are	Use abbreviations for quantities unless they were used in the question or you defined them (either in a statement or a diagram)
Let *a* be the number of dogs Alfie had	Let Alfie be *a*
Let *m* be the mass in kilograms	Let *m* be kilograms
Let the area of the base be *b*	Let the base be *b*
Use single-letter abbreviations, eg. Let the number of Elephants be *n*	Let the number of elephants be *noe*
In general, lower case letters are preferable	Let the amount Judy had be *J*
Use letters which will remind the reader of what they stand for: let Sam's height be *s* and Mary's *m*.	Use letters that have no relationship with the question eg. Let Sam's height be *p* and Mary's height be *q*

Use mathematical notation in the standard way	
Do	**Don't**
$\int 4x\ dx$	$\int 4x$
$\int (4x + 7)dx$	$\int 4x + 7\ dx$

Diagrams and Graphs	
Do	**Don't**
Use diagrams to help you clarify a question	Try to do complex questions in your head – most people cannot
Label your diagrams with relevant dimensions, angles etc	Draw un-labelled sketches that may or may not have any relevance to the question
Label graphs and axes. For graphs of algebraic functions, write the function on the graph	Assume that your teacher will understand your graph if it is not labelled

Proof

At Levels 5 and 6, you will sometimes be asked to prove a result that you already know or that is given. Proving means demonstrating that the result ***must*** be true.

Good communication will sometimes constitute a proof, though proofs are generally a bit more formal than good communication.

As explained in '*What is Maths*' the idea of proof is fundamental to the building of the body of mathematical knowledge.

Note that proof in a court of law is 'proof beyond reasonable doubt'.
Mathematical proof is proof beyond any doubt, proof with absolute certainty.

Proof is introduced in Modules A5-8 (Algebraic Proofs) and G5-1 (Geometric Proofs). The proofs dealt with in those modules are proof by deduction. There are other types of proof: proof by exhaustion, proof by contradiction and proof by induction. These are not required for Australian Years 7-10 Maths or for Maths Methods. They are however required for Specialist Maths and are included in the M1 Maths Module A6-5 (Further Methods of Proof).