1999 TOOWOOMBA MATHEMATICS TEAM CHALLENGE

SENIOR SECONDARY

TEAMS CONTEST

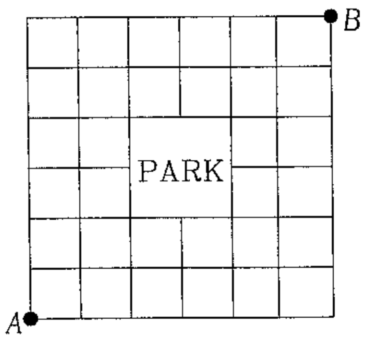
Time: *45 minutes*
Calculators may be used
Total of 100 points—Points as shown

T1 (8 points)

The number 1867 is multiplied by a positive integer k. The last four digits of the product are 1999. Find the smallest possible value of k.

T2 (8 points)

A road map of Grid City is shown in the diagram. The perimeter of the park is a road but there is no road through the park. How many different *shortest* road routes are there from point A to point B?



T3 (9 points)

In an arithmetic sequence $t_1 = 98$ and $t_{13} = 89$. Define $T = t_n + t_{n+1} + t_{n+2} + \cdots + t_{n+6}$. Determine the value of n that produces the minimum value of $|T|$, where $|T|$ represents the absolute value of T.

T4 (10 points)

Starting from opposite ends of a straight track at the same time, two runners begin jogging at constant rates. Each person runs to the far end of the track and back to his starting position. Their first meeting is 600 m from one end and, after both turning around, their second meeting is 300 m from the other end. How long is the track in metres?

T5 (10 points)

One of the floats in the Toowoomba Flower Festival Parade held the Mayor and all Council members. At the end of the parade, everyone shook hands with everyone else. A straggler suddenly arrived and shook hands with only those people whom the straggler knew. Altogether, there were 141 handshakes. How many people were on the float?

T6 (10 points)

Ann and Bob play a game in which Ann starts by rolling a fair die and then Bob tosses a fair coin. They repeat this alternating pattern until one of them wins. Ann wins if a "six" occurs and Bob wins if a "head" occurs. What is the probability that Ann wins the game?

T7 (10 points)

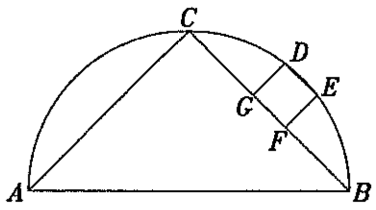
Determine the number of ways that 5 different prizes can be awarded to 4 students so that each student receives at least one prize and all prizes are awarded.

T8 (11 points)

In a 14 team baseball league, each team played each of the other teams 10 times. At the end of the season, the number of games won by each team differed from those won by the team that immediately followed it by the same amount. Determine the greatest number of games the last place team could have won, assuming that no ties occurred.

T9 (12 points)

AB is the diameter of a semicircle of radius one. C, D, and E are points on the semicircle. F and G are points on BC. Given $\triangle ABC$ is isosceles and $DEFG$ is a square, calculate the area of $DEFG$.



T10 (12 points)

Determine all pairs of integers (x, y) which satisfy the equation

$$6x^2 - 3xy - 13x + 5y = -11.$$

1999 TOOWOOMBA EDUCATION CENTRE
MATHEMATICS TEAM CHALLENGE

SENIOR TEAMS CONTEST

ANSWERS

Question	Mark	Answer
T1	8	2797
T2	8	524
T3	9	129
T4	10	1500
T5	10	17
T6	10	2/7 (or 0.2857)
T7	10	240
T8	11	52
T9	12	0.08 (or 2/25)
T10	12	(1, -2), (2, 9)

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ANSWERS

Question	Mark	Answer
T1	8	
T2	8	
T3	9	
T4	10	
T5	10	
T6	10	
T7	10	
T8	11	
T9	12	
T10	12	