2005 TOOWOOMBA MATHEMATICS TEAM CHALLENGE

SENIOR SECONDARY

RELAY CONTEST

Time: 1 hour
Calculators may be used
Each question is worth 5 points
Total of 100 points

R1 (3 points) (97 points remaining)

"My age is curious", said Alice. "In fact my age in years is one more than three times the sum of the digits of my age." What is Alice's age, in years?

R2 (3 points) (94 points remaining)

The relationship between Fahrenheit (F) and Celsius (C) temperature scales is given by the formula

$$F = \frac{9}{5}C + 32.$$

At what temperature do the two scales give the same numerical number of degrees?

R3 (3 points) (91 points remaining)

The horizontal cross section of a rectangular swimming pool measures 12×5 metres. Due to evaporation, the water level dropped 20 cm and so a hose with a flow rate of 200 litres per hour is to be used to refill the pool. Ignoring further evaporation, how many hours will it take to refill the pool?

R4 (4 points) (87 points remaining)

A 300 hitter in softball gets a safe hit each time at bat with probability 0.3. If a 300 hitter bats four times in a game, what is the probability that she gets at least one safe hit.

R5 (4 points) (83 points remaining)

How many of the first one hundred positive integers are divisible by all of the numbers 2,3,4, and 5?

R6 (4 points) (79 points remaining)

Suppose that a, b, and c are positive integers such that $\frac{a}{3} = \frac{b}{4} = \frac{c}{5}$, and $abc = 1620$. Find the value of b.

R7 (5 points) (74 points remaining)

A bicycle with 26 inch diameter wheels is travelling at 50 km/h along a straight road. Relative to the road, what is the tangential speed (in km/h) of a point on the top of a wheel?

R8 (5 points) (69 points remaining)

Fibonacci's famous sequence of numbers is

$$1, 1, 2, 3, 5, 8, 13, 21, \cdots$$

where each number, after the first two numbers, is the sum of its preceding two numbers. Angus announced that he tried starting with two other numbers and used the same rule that each successive number is the sum of the preceding two numbers. Including his chosen two numbers, he found that the eighth and ninth numbers in his sequence were Fibonacci's starting numbers 1 and 1. Find the first number in Angus's sequence.

R9 (5 points) (64 points remaining)

If $f(x) = \sqrt{x}$ and $g(x) = (x+1)^2$ and $h(x) = 2x + 1$, find the value of the composite function $f(g(h(0.5)))$.

R10 (5 points) (59 points remaining)

Three six sided dice are thrown. Find the probability that the three resulting numbers can be arranged as three consecutive integers.

R11 (5 points) (54 points remaining)

How many integers greater than ten and less than one hundred, are increased by nine when their digits are reversed?

R12 (5 points) (49 points remaining)

Let $x_1 = 101$, and for $n > 1$ let $x_n = \frac{n}{x_{n-1}}$. Find the value of the product

$$x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9 x_{10} x_{11} x_{12}.$$

R13 (5 points) (44 points remaining)

Find the integer value of

$$\frac{1234567890}{(1234567890)(1234567892) - 1234567891^2}$$

R14 (5 points) (39 points remaining)

Due to wear, a spherical ball-bearing has its volume evenly reduced by 27.1%. Find the percentage reduction in its surface area. (If you must know, $V = \frac{4}{3}\pi r^2$, $A = 4\pi r^2$.)

R15 (6 points) (33 points remaining)

Mr McGregor's shed can hold all his bales of wool except for eight bales. If the shed were half again as big, it could then hold eight more bales than he owns. What is the number of bales of wool owned by Mr McGregor?

R16 (6 points) (27 points remaining)

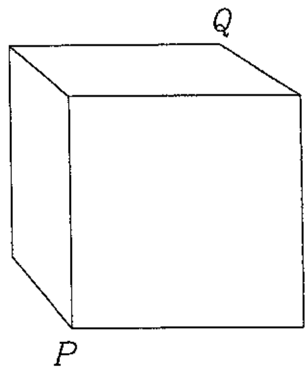
In a geometric sequence of positive numbers, the sum of the first two terms is 51 and the sum of the first four terms is 255. What is the first term?

R17 (6 points) (21 points remaining)

How many ordered pairs of integers (m, n) satisfy the equation $m^2 + 4m + n^2 = 9$?

R18 (7 points) (14 points remaining)

In the figure, PQ is the main diagonal of a cube and PQ has length equal to 1. Find the surface area of the cube.



R19 (7 points) (7 points remaining)

What is the smallest value of $x^2 + y^2$ on the line $3y + 4x = 12$?

R20 (7 points) (0 points remaining)

A supermarket has 128 crates of apples. Each crate contains at least 120 apples and at most 144 apples. What is the largest integer n such that there must be at least n crates containing the same number of apples?

MATHS TEAMS CHALLENGE (2005)
Relay Answer sheet
SENIOR SECONDARY

Question	Answer	Attempts x or /							Score	Progressive Score
		7	6	5	4	3	2	1		
R1 (3 points)	13									
R2 (3 points)	-40									
R3 (3 points)	60									
R4 (4 points)	0.7599									
CHANGE										
R5 (4 points)	1									
R6 (4 points)	12									
R7 (5 points)	100									
R8 (5 points)	-8									
CHANGE										
R9 (5 points)	3									
R10 (5 points)	$^1/_9$ or 0.1111…									
R11 (5 points)	8									
R12 (5 points)	46080									
CHANGE										
R13 (5 points)	-1234567890									
R14 (5 points)	19									
R15 (6 points)	40									
R16 (6 points)	17									
CHANGE										
R17 (6 points)	8									
R18 (7 points)	2									
R19 (7 points)	5.76 or $^{144}/_{25}$									
R20 (7 points)	6									
									TOTAL	

School: ______________________________

Team 1: ☐ Team 2: ☐