

# 2003 TOOWOOMBA MATHEMATICS TEAM CHALLENGE

## SENIOR SECONDARY

### RELAY CONTEST

*Time: 1 hour*

*Calculators may be used*

*Total of 100 points*

**R1** (4 points)

(96 points remaining)

What are the values of  $x$  such that  $(2x + 1)^2 = 9$ ?

**R2** (4 points)

(92 points remaining)

Given a standard deck of 52 playing cards, what is the fewest cards you would have to draw in order to be certain that you have three of a kind; that is, three kings, or three eights, etc?

**R3** (4 points)

(88 points remaining)

Consider a circular target of radius one. An arrow is fired at the target. Given that the arrow hits the target, and has an equal chance of hitting any point within the circle, what is the probability that it hits closer to the centre of the circle than to the circumference?

**R4** (4 points)

(84 points remaining)

Let  $\lfloor x \rfloor$  represent the largest integer which is less than or equal to  $x$ . For example  $\lfloor 3 \rfloor = 3$ ,  $\lfloor 2.6 \rfloor = 2$ . If  $x$  is positive and  $x \lfloor x \rfloor = 19$ , what is the value of  $x$ ?

**R5** (4 points)

(80 points remaining)

An automobile with five tyres (four tyres and a spare) travelled 30000 km. All five tyres were used equally. How many kilometres did each tyre go?

**R6** (4 points)

(76 points remaining)

If  $P = 3^{2003} + 3^{-2003}$  and  $Q = 3^{2003} - 3^{-2003}$ , find the value of  $P^2 - Q^2$ .

**R7** (5 points)

(71 points remaining)

When a positive integer  $N$  is divided by 60, the remainder is 49. When  $N$  is divided by 15, what is the remainder?

**R8** (5 points)

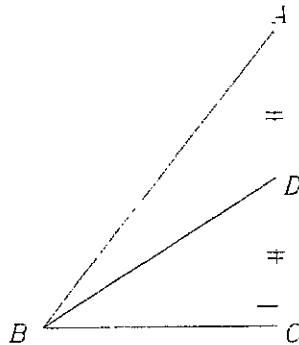
(66 points remaining)

How many two-digit numbers are divisible by 2 or 7?

**R9** (5 points)

(61 points remaining)

In the diagram,  $AD = DC$ ,  $\sin \angle DBC = 0.6$  and  $\angle ACB = 90^\circ$ . What is the value of  $\tan \angle ABC$ ?



**R10** (5 points)

(56 points remaining)

A regular hexagon is a six-sided figure that has all of its angles equal and all its sides lengths equal. If  $P$  and  $Q$  are points on a regular hexagon that has a side length of one, what is the maximum possible length of the line segment  $PQ$ ?

**R11** (5 points)

(51 points remaining)

In how many ways can 75 be expressed as the sum of two or more consecutive integers?

**R12** (5 points)

(46 points remaining)

What is the coefficient of  $x^{99}$  (i.e., the number in front of  $x^{99}$ ) in the expansion of

$$(x - 1)(x - 2)(x - 3) \dots (x - 100)$$

**R13** (5 points)

(41 points remaining)

What is the value of  $x$  such that  $\log_2(\log_2(2x - 2)) = 2$ ?

**R14** (5 points)

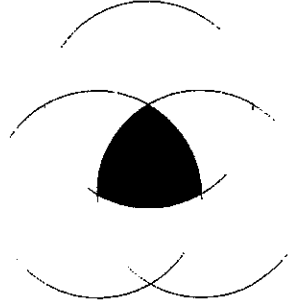
(36 points remaining)

A straight one-way street has eight consecutive traffic lights. Every light remains green for 90 seconds, amber for 3 seconds and red for 90 seconds. The lights are synchronised so that each light turns red 10 seconds after the preceding one turns red. What is the longest interval of time, in seconds, during which all lights are green?

**R15** (6 points)

(30 points remaining)

Three circles, each of radius one, intersect such that each circle passes through the centres of the other two circles. What is the area of the shaded region, accurate to 3 decimal places?



**R16** (6 points)

(24 points remaining)

Lines are concurrent if they each pass through the same point. The lines  $y = 2x + 3$ ,  $y = 5x + 9$ , and  $y = 8x + b$  are concurrent. What is the value of  $b$ ?

**R17** (6 points)

(18 points remaining)

The probability that event  $A$  occurs is  $3/4$ ; the probability that event  $B$  occurs is  $2/3$ . Let  $p$  be the probability that both  $A$  and  $B$  occur. What are the smallest and largest possible values of  $p$ ?

**R18** (6 points)

(12 points remaining)

Cecilia leaves school at the same time each day. If she cycles at 20 km/hr, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/hr, she arrives home at 5:15 in the afternoon. At what speed, in km/hr, must she cycle to arrive home at 5:00 in the afternoon?

**R19** (6 points)

(6 points remaining)

Each of the six sides of a cube is coloured black or white. In how many ways can this be done (disregarding those that are the same after rotation)?

**R20** (6 points)

(0 points remaining)

A lattice point in 3-space is a point having three integer coordinates. How many lattice points lie wholly within a sphere with centre  $(0, 0, 0)$  and radius 3?

**MATHS TEAMS CHALLENGE (2003)**  
**Relay Answer sheet**  
**SENIOR SECONDARY**

| Question       | Answer    | Attempts x or / |   |   |   |   |   |   | Score        | Progressive Score |
|----------------|-----------|-----------------|---|---|---|---|---|---|--------------|-------------------|
|                |           | 7               | 6 | 5 | 4 | 3 | 2 | 1 |              |                   |
| R1 (4 points)  | 1, -2     |                 |   |   |   |   |   |   |              |                   |
| R2 (4 points)  | 27        |                 |   |   |   |   |   |   |              |                   |
| R3 (4 points)  | 0.25      |                 |   |   |   |   |   |   |              |                   |
| R4 (4 points)  | 4.75      |                 |   |   |   |   |   |   |              |                   |
| <b>CHANGE</b>  |           |                 |   |   |   |   |   |   |              |                   |
| R5 (4 points)  | 24000     |                 |   |   |   |   |   |   |              |                   |
| R6 (4 points)  | 4         |                 |   |   |   |   |   |   |              |                   |
| R7 (5 points)  | 4         |                 |   |   |   |   |   |   |              |                   |
| R8 (5 points)  | 51        |                 |   |   |   |   |   |   |              |                   |
| <b>CHANGE</b>  |           |                 |   |   |   |   |   |   |              |                   |
| R9 (5 points)  | 1.5       |                 |   |   |   |   |   |   |              |                   |
| R10 (5 points) | 2         |                 |   |   |   |   |   |   |              |                   |
| R11 (5 points) | 5 or 10   |                 |   |   |   |   |   |   |              |                   |
| R12 (5 points) | -5050     |                 |   |   |   |   |   |   |              |                   |
| <b>CHANGE</b>  |           |                 |   |   |   |   |   |   |              |                   |
| R13 (5 points) | 9         |                 |   |   |   |   |   |   |              |                   |
| R14 (5 points) | 20        |                 |   |   |   |   |   |   |              |                   |
| R15 (6 points) | 0.705     |                 |   |   |   |   |   |   |              |                   |
| R16 (6 points) | 15        |                 |   |   |   |   |   |   |              |                   |
| <b>CHANGE</b>  |           |                 |   |   |   |   |   |   |              |                   |
| R17 (6 points) | 5/12, 2/3 |                 |   |   |   |   |   |   |              |                   |
| R18 (6 points) | 12        |                 |   |   |   |   |   |   |              |                   |
| R19 (6 points) | 8         |                 |   |   |   |   |   |   |              |                   |
| R20 (6 points) | 93        |                 |   |   |   |   |   |   |              |                   |
|                |           |                 |   |   |   |   |   |   | <b>TOTAL</b> |                   |

School: \_\_\_\_\_

Team 1:  Team 2: