2000 TOOWOOMBA MATHEMATICS TEAM CHALLENGE

SENIOR SECONDARY

TEAMS CONTEST

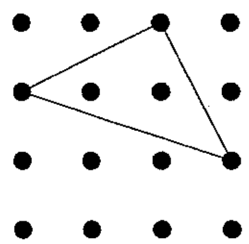
Time: *45 minutes*
Calculators may be used
Total of 100 points—Points as shown

T1 (8 points)

Solve $|x - |2x + 1|| = 3$.

T2 (9 points)

Consider a square grid consisting of 4×4 points. How many triangles are there with vertices on the points? (An example triangle is shown in the figure. The three vertices may not lie on a straight line.)



T3 (10 points)

Consider the points 1, 1/2, 1/3, ... on the real number line. You are given five small bars, all of length ℓ, which are to be placed on the number line such that all the points will be covered. What is the minimum value of ℓ that will allow you to do this?

T4 (10 points)

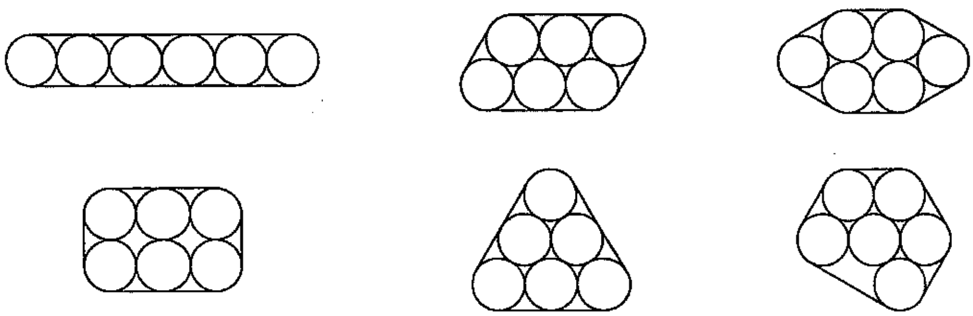
A square and a circle are placed such that the centre of the circle and the centre of the square coincide. The region inside the square but outside the circle has the same area as the region inside the circle but outside the square. What is the ratio d/s of the diameter of the circle to the length of the sides of the square? Write your answer accurate to 4 significant figures.

T5 (12 points)

Through an informer from the underworld, the police knows the meeting place of a gang. The identity of the different gang members, however, is unknown. It is the duty of a policeman to shadow the leader of the gang. The policemen knows that this leader is tallest of the five persons, all of whom have different heights. After the meeting, the gangsters, as a safety measure, leave the building separately at intervals of 5 minutes. As the policeman cannot see who is tallest, he decides to let the first two gangsters go and to shadow the first one after that who is taller than all those who left before. What is the probability that the policeman will shadow the correct person?

T6 (10 points)

Six soft drink cans are bound together by a non-elastic strap. Six methods of doing this are shown. Out of these six methods determine the length of strap needed for the method(s) which require the smallest length (disregarding any overlap at each end). Assume each can has radius 1. Give your answer accurate to 6 significant figures.

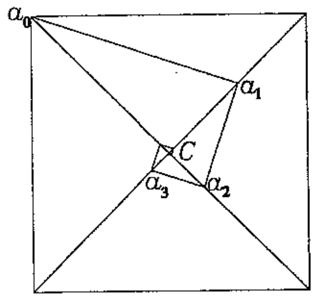


T7 (10 points)

The natural numbers are summed in groups as follows: 1, $2+3$, $4+5+6$, $7+8+9+10$, and so on. What is the sum of the 100th group?

T8 (10 points)

Consider a square with centre c and with a diagonal of length 2. Construct a spiral $a_0a_1a_2\ldots$ as shown. The distance from a_{n+1} to c is half the distance from a_n to c. What is the total length of the spiral? Give your answer accurate to 5 significant figures.

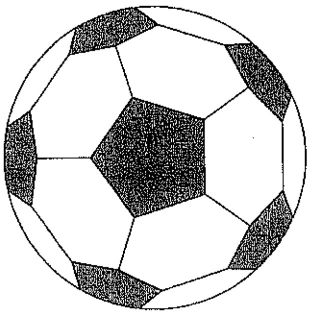


T9 (12 points)

Let $p(x) = x^6 + ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ be a polynomial such that $p(1) = 1, p(2) = 2, p(3) = 3$, $p(4) = 4$, $p(5) = 5$, and $p(6) = 6$. What is $p(7)$?

T10 (10 points)

A soccer ball is a polyhedron that has 32 faces which are either regular pentagons or regular hexagons. How many edges does a soccer ball have?



TOOWOOMBA EDUCATION CENTRE
2000 MATHEMATICS TEAM CHALLENGE

SENIOR TEAMS CONTEST

ANSWERS

Question	Points	Answer
T1	7	2, -4/3
T2	9	516
T3	10	1/10
T4	10	1.128
T5	12	13/30 or 0.433
T6	10	17.7473
T7	10	500050
T8	10	2.2361
T9	12	727
T10	10	90

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MATHEMATICS TEAM CHALLENGE 2000

Solutions – Senior Secondary Teams Contest

T1. $|x - |2x+1|| = 3 \Rightarrow$ either $x - |2x+1| = 3$ ① or $x - |2x+1| = -3$ ②

Case ① $x - 3 = |2x+1| \geq 0 \Rightarrow x \geq 3$ & $x - 3 = 2x + 1 \Rightarrow x \geq 3$ & $x = -4$???

Case ② $x + 3 = 2x + 1 \geq 0 \Rightarrow$ either $x \geq -3$ & $2x+1 \geq 0$ & $2x + 1 = x + 3$

or $x \geq -3$ & $2x + 1 < 0$ & $-2x - 1 = x + 3$

$\Rightarrow$ either $x \geq -3$ & $x \geq -\frac{1}{2}$ & $\underline{\underline{x = 2}}$ ✓

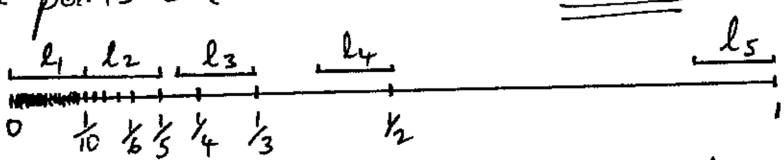
or $x \geq -3$ & $x < -\frac{1}{2}$ & $\underline{\underline{x = -\frac{4}{3}}}$ ✓

(Alternatively a graphical approach might be used)

T2. No of triangles $= \binom{16}{3} - \binom{4}{3} \times (4 + 4 + 2) + 4$

$= 560 - 44 = \underline{\underline{516}}$

T3 All the points can be covered with $\underline{\underline{l = \frac{1}{10}}}$ as follows:



l_5 covers 1; l_4 covers $\frac{1}{2}$; l_3 covers $\frac{1}{3}, \frac{1}{4}$; l_4 covers $\frac{1}{5}, \frac{1}{6} \cdots \frac{1}{9}$; l_5 covers $\frac{1}{10}, \frac{1}{11} \cdots$

Can l be reduced?

Suppose $\frac{1}{10} > l > \frac{1}{11}$ then l_5, l_4, l_3 must remain as above,

l_1 must cover $\frac{1}{11}, \frac{1}{12}, \frac{1}{13}, \ldots$ & l_2 must cover $\frac{1}{5}, \frac{1}{6}, \ldots, \frac{1}{10}$.

But $\frac{1}{5} - \frac{1}{10} = \frac{1}{10} > l$ so l_2 fails.

T4. By construction, Area ○ = Area □. $\therefore \pi\left(\frac{d}{2}\right)^2 = s^2 \Rightarrow \frac{d}{s} = \frac{2}{\sqrt{\pi}} = \underline{\underline{1.128}}$

T5. Assume $A > B > C > D > E$. Total no of orders $= 5! = 120$

Arrangements such that A is correctly shadowed:

_ _ A _ _, _ _ E A _, _ _ D A _, _ _ C A E, _ _ C A D, B _ _ _ A, _ B _ _ A

$\# = 4! + 3! + 3! + 2! + 2! + 3! + 3!$

$= 52 \quad \therefore$ req'd prob $= \frac{52}{120} = \underline{\underline{\frac{13}{30}}} = 0.433$

T6. For method 1, $l = 2\pi + 20$. For methods 2 - 5, $l = 2\pi + 12 = 18.2832$

For method 6, $l = 2\pi + 8 + 2\sqrt{3} = \underline{\underline{17.7473}}$

T7. $S_1 = 1$ (first term 1), $S_2 = 2 + 3$ (first term $1+1$), $S_3 = 4 + 5 + 6$ (first term $1+1+2$), $S_4 = 7 + 8 + 9 + 10$ (first term $1+1+2+3$) ...

$\therefore S_{100}$ has 1st term $(1 + 1 + 2 + 3 + \cdots + 99) = \frac{99 \times 100}{2} + 1 = 4951$

$\therefore S_{100} = 4951 + 4952 + \cdots + 5050 = \left(\frac{4951 + 5050}{2}\right) \times 100 = \underline{\underline{500050}}$

T8. By Pythag. $\overline{a_0 a_1}^2 = 1^2 + \frac{1}{2}^2 = \frac{5}{4}$, $\overline{a_1 a_2}^2 = \frac{1}{2}^2 + \frac{1}{4}^2 = \frac{5}{16}$, ...

$\therefore l = \sqrt{\frac{5}{4}} + \sqrt{\frac{5}{16}} + \sqrt{\frac{5}{64}} + \cdots = \sqrt{5}\left(1 + \frac{1}{4} + \frac{1}{8} + \cdots\right) = \frac{\sqrt{5} \times \frac{1}{2}}{1 - \frac{1}{2}} = \sqrt{\ } = \underline{\underline{2.2361}}$

T9. Put $f(x) = p(x) - x$. Then $f(1) = f(2) = f(3) = f(4) = f(5) = f(6) = 0$

$\Rightarrow f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$

$\therefore f(7) = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

$\therefore p(7) = f(7) + 7 = \underline{\underline{727}}$

T10. 32 faces = 12 pentagons + 20 hexagons

\# edges $= 12 \times 5 + \frac{20 \times 3}{2} = \underline{\underline{90}}$
