

## Listen Maths – Probability

### The Basic Ideas of Probability

Probability is a number between 0 and 1 that tells us specifically the likelihood of something happening.

There are three ways to find probability: data, indifference and guesswork.

I'll say that again: There are three ways to find probability: data, indifference and guesswork.

Let's explain.

If you ask, 'how **likely** is it that my matchbox will land on its end if I drop it?', the answer might be something like, 'Not very likely', or 'less likely than landing flat', or 'almost impossible'.

If you ask what is the **probability** that my matchbox will land on its end if I drop it, then the answer should be a number, a fraction between 0 and 1.

Likelihood is a description. Probability is a number.

Probability is the fraction of times something would happen in the long run.

That's important to remember, so here it is again:

Probability is the fraction of times something would happen in the long run.

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So the probability that my matchbox will land on its end is the fraction of times that it would land on its end in the long run, i.e. if I dropped it a lot of times.

If I drop it 10 times and it lands on its end once, I can say that the probability is  $1/10$  or 0.1 or 10%.

But if I do the experiment again and drop it another 10 times, there's no guarantee it would land on its end once. It might happen twice or not at all. It could happen 3 times

So my 10% is just a rough estimate of the probability.

If I did the experiment again, this time with 100 trials (i.e. dropping it 100 times), I might find 6 successes – a success being the outcome I am interested in, in this case, landing on its end.

This would give me a more reliable estimate of the probability –  $6/100$  or 0.06 or 6%.

It still might not be quite right though – the next time I do the experiment with 100 trials, I could get 5 successes or 8. But I'm not likely to get 30, so I would be fairly sure that the probability is somewhere around 6% - maybe between 4% and 9%.

I could get an even more reliable result if I dropped it 1000 times. If I get 72 successes, this would suggest a probability of  $72/1000$  or 0.072 or 7.2%.

The more times I try it, the more reliable my result would be. If I did an infinite number of trials, I would get the exact probability, maybe something like 0.0712853 bla bla bla.

So probability is actually the fraction of times something would happen if we tried it an infinite number of times. But we say 'in the long run' for short. We say 'Probability is the fraction of times something would happen in the long run'.

The fraction of times the matchbox would land on its end in the long run is 0.0712853 bla bla bla, and so the probability of it landing on its end is 0.0712853 bla bla bla.

But, of course we could never drop it an infinite number of times.

If we use an experiment to find the probability of something happening, we have to do less than infinity trials, so our result will always be an approximation. The more trials we do, though, the better the approximation.

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There are a few special words that we use to describe probability experiments:

Each drop of the matchbox to see what happens is called a trial.

An experiment consists of a series of trials.

Each trial has a number of possible outcomes, in this case landing on its end, landing on its edge, and landing flat.

The outcome we are interested in, in this case landing on its end, is called a success.

The fraction of successes, i.e. the number of successes divided by the number of trials, is called the relative frequency.

The relative frequency is our approximation for the probability. The more trials the better the approximation.

Those words are important to know, so I'll repeat them, giving the meaning first, then the word, with a gap for you to say it before me.

Each drop of the matchbox is called . . . a trial

A series of trials is called . . . an experiment

In each trial, there are a number of possible . . . outcomes.

The outcome we are interested in is called . . . a success.

The fraction of successes in the experiment, i.e. the number of successes divided by the number of trials is called . . . the relative frequency.

The relative frequency is an approximate value for . . . the probability.

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A probability experiment generates data, in this case the number of times we drop the matchbox and the number of times it lands on its end. We then use that data to estimate the probability.

But we can sometimes get the data without doing an experiment.

Suppose we want to know the probability that someone who gets yellow fever will die from it. To find out by doing an experiment, we would need to infect say 100 people with yellow fever and see how many die.

However, such experiments tend to be frowned upon.

But data already exists in health records in countries that have yellow fever. This is called pre-existing data.

Suppose we find that, in Panama, 2592 people have been recorded as getting yellow fever, and, of those, 487 died from it. The relative frequency is then  $487 \div 2592$ , which comes to about 0.19 or 19%. This relative frequency is our estimate of the probability. So we can say that the probability of dying from yellow fever is about 19%.

When using pre-existing data, as with experiments, the more data we have, the more reliable our estimation of the probability.

To summarise, probability can be estimated by collecting data. We can generate the data with an experiment, or we can find pre-existing data.

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Sometimes we can find probabilities exactly without any data. We use indifference.

For instance, if I toss a coin, I know that the probability of getting a head is  $\frac{1}{2}$  or 0.5 or 50%. How?

Indifference doesn't mean that we don't care. It means that there's no difference between the possible outcomes that could make any one any more or less likely than any other. In other words, we know that all possible outcomes are equally likely. They are indifferent, that is: not different.

If we toss a coin, there is no difference between the two sides of the coin that will make it more likely to land heads or tails. In other words, in the long run, it will land just as many times each way. It will land heads 50% of the times and tails 50% of the times in the long run.

This means that the probability of a head is 50% and the probability of a tail is 50%.

Of course, we couldn't use indifference with the matchbox because there is an obvious difference between the sides of a matchbox that makes it more likely to land flat than to land on its end.

Note that the patterns on the head and tail of a coin are different, but their weighting is generally about the same. The different patterns might cause the probability of heads to be very slightly off 50%, maybe 50.0001%, but we would have to do billions of trials to notice that. We assume the probability is 50%.

We can use the idea of indifference in other situations, like with a game spinner or with a die (die is the singular of dice). All six sides of a die are essentially the same and will land upwards the same number of times in the long run –  $\frac{1}{6}$  of the times each. So the probability of getting any number on a die is  $\frac{1}{6}$ . A 6 is just as likely as a 2.

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So now, we have seen two methods to find a probability – data and indifference.

But, there are situations where we cannot use either of these methods.

For instance, suppose we want to know the probability that Grandma will break her leg if she falls out of the window.

There's no pre-existing data because Grandma has never fallen out of the window. We probably can't do an experiment and push her out 100 times to see how many times she breaks her leg because she probably wouldn't co-operate. And there's no reason to assume that breaking her leg and not breaking her leg are equally likely, so we can't use indifference either.

In situations like this, we can still talk about the likelihood, but the mathematical idea of probability can't strictly be applied.

It may still be possible to come up with a number though. By guesswork.

If we say that the probability of Grandma breaking her leg is 20%, then we are really just saying that we think it's about as likely that Grandma will break her leg as that something with a probability of 20% will happen.

I'll say that again: we think it's about as likely that Grandma will break her leg as that something with a probability of 20% will happen.

If it's a ground floor window, we might guess 20%; if it's an upstairs window, we might guess 60%; if it's a window on the 20<sup>th</sup> floor, we might guess 95%.

What's the probability that there is life after death? We can't try that even once, so guessing is definitely the only option. A religious person might guess 90% or maybe even 100%. An atheist might guess 10%. Guesses are always subjective – different for different people depending on what they know and believe.

So, to summarise, there are three ways to find a probability:

1 using data (from an experiment or pre-existing)

2 using indifference

and 3 guessing.

These ideas are explained in more detail in [M1Maths.com](http://M1Maths.com). Click Modules, then Probability, then Level 1, then P1-1.