

## Tables showing which M1 Maths modules relate to each Queensland Years 11-12 Mathematical Methods topic

<b>Unit 1</b>	<a href="#">Topic 1</a>	<a href="#">Topic 2</a>	<a href="#">Topic 3</a>	<a href="#">Topic 4</a>	<a href="#">Topic 5</a>	
<b>Unit 2</b>	<a href="#">Topic 1</a>	<a href="#">Topic 2</a>	<a href="#">Topic 3</a>	<a href="#">Topic 4</a>	<a href="#">Topic 5</a>	<a href="#">Topic 6</a>
<b>Unit 3</b>	<a href="#">Topic 1</a>	<a href="#">Topic 2</a>	<a href="#">Topic 3</a>			
<b>Unit 4</b>	<a href="#">Topic 1</a>	<a href="#">Topic 2</a>	<a href="#">Topic 3</a>	<a href="#">Topic 4</a>	<a href="#">Topic 5</a>	

The syllabus element is in the left column and the relevant module is in the right column.

### Unit 1 Topic 1 – Arithmetic and Geometric Sequences and Series 1

#### Arithmetic Sequences (4 hours)

Recursive definition of an arithmetic sequence: $tn+1 = tn+d$	A6-2 Arithmetic Sequences
The formula $tn = t1+(n-1)d$ for the general term of an arithmetic sequence and its linear nature	
Arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest	
Sum of the first $n$ terms of an arithmetic sequence: $S_n = n/2(2t_1+(n-1)d) = n/2(t_1+t_n)$	

### Unit 1 Topic 2 – Functions and Graphs

#### Functions (4 hours)

a relation as a mapping between sets, a graph and as a rule or a formula that defines one variable quantity in terms of another	A3-7 Functions
recognise the distinction between functions and relations and use the vertical line test to determine whether a relation is a function	
use function notation, domain and range, and independent and dependent variables	A1-2 Relations 2 A3-6 Domain and range A3-7 Functions
examine transformations of the graphs of $f(x)$ , including dilations and reflections, and the graphs of $y = af(x)$ and $y = f(bx)$ , translations, and the graphs of $y = f(x+c)$ and $y = f(x)+d$ ; $a,b,c,d \in R$	A5-9 Algebraic transformations

recognise and use piece-wise functions as a combination of multiple sub-functions with restricted domains	A5-10 Further relations
identify contexts suitable for modelling piece-wise functions and use them to solve practical problems (taxation, taxis, the changing velocity of a parachutist).	
<b>Quadratic Relationships (7 hours)</b>	
examine examples of quadratically related variables	A4-2 Quadratic functions
recognise and determine features of the graphs of $y=x^2$ , $y = ax^2+bx+c$ , $y = a(x-b)^2+c$ , and $y = a(x-b)(x-c)$ , including their parabolic nature, turning points, axes of symmetry and intercepts	
solve quadratic equations algebraically using factorisation, the quadratic formula (both exact and approximate solutions), and completing the square and using technology	
identify contexts suitable for modelling with quadratic functions and use models to solve problems with and without technology; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis	
understand the role of the discriminant to determine the number of solutions to a quadratic equation	
determine turning points and zeros of quadratic functions with and without technology.	
<b>Inverse Proportions (3 hours)</b>	
examine examples of inverse proportion	N3-3 Proportion A3-9 Reciprocal functions
recognise features of the graphs of $y = 1/x$ and $y=a/(x-b)$ , including their hyperbolic shapes, their intercepts, their asymptotes and behaviour as $x \rightarrow \infty$ and $x \rightarrow -\infty$ .	
<b>Powers and Polynomials (9 hours)</b>	
identify the coefficients and the degree of a polynomial	A5-1 Polynomial functions A4-1 Factorising
expand quadratic and cubic polynomials from factors	
recognise and determine features of the graphs of $y=x^3$ , $y=a(x-b)^3+c$ and $y=k(x-a)(x-b)(x-c)$ , including shape, intercepts and behaviour as $x \rightarrow \infty$ and $x \rightarrow -\infty$	
use the factor theorem to factorise cubic polynomials in cases where a linear factor is easily obtained	
solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained	
recognise and determine features of the graphs $y = a(x-b)^4+c$ , including shape and behaviour	A3-5 Solving by graphing
solve equations involving combinations of the functions above, using technology where appropriate.	

<b>Graphs of Relations (3 hours)</b>	
recognise and determine features of the graphs of $x^2+y^2=r^2$ and $(x-a)^2+(y-b)^2=r^2$ , including their circular shapes, centres and radii	A5-10 Further relations
recognise and determine features of the graph of $y^2=x$ , including its parabolic shape and axis of symmetry	

## Unit 1 Topic 3 – Counting and Probability

### Language of Events and Sets (4 hours)

recall the concepts and language of outcomes, sample spaces and events as sets of outcomes	P1-1 Probability P2-1 Compound events
use set language and notation for events, including $\bar{A}$ or $A'$ for the complement of an event, $A \cap B$ for the intersection of events $A$ and $B$ , and $A \cup B$ for the union, and recognise mutually exclusive events	P2-3 Venn diagrams
use everyday occurrences to illustrate set descriptions and representations of events, and set operations, including the use of Venn diagrams.	

### Review of the Fundamentals of Probability (3 hours)

recall probability as a measure of 'the likelihood of occurrence' of an event	P1-1 Probability P2-1 Compound events
recall the probability scale: $0 \leq P(A) \leq 1$ for each event $A$ , with $P(A)=0$ if $A$ is an impossibility and $P(A)=1$ if $A$ is a certainty	
recall the rules $P(\bar{A})=1-P(A)$ and $P(A \cup B)=P(A)+P(B)-P(A \cap B)$	
use relative frequencies obtained from data as point estimates of probabilities.	

### Conditional Probability and Independence (7 hours)

understand the notion of a conditional probability, and recognise and use language that indicates conditionality	P4-1 Complex probabilities
use the notation $P(A B)$ and the formula $P(A \cap B)=P(A B)P(B)$ to solve problems	
understand and use the notion of independence of an event $A$ from an event $B$ , as defined by $P(A B)=P(A)$	P2-1 Compound events
establish and use the formula $P(A \cap B)=P(A)P(B)$ for independent events $A$ and $B$	
use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events.	

<b>Binomial Expansion (3 hours)</b>	
understand the notion of a combination as an unordered set of $r$ objects taken from a set of $n$ distinct objects	A6-1 Combinations and the binomial Expansion
recognise and use the link between Pascal's triangle and the notation $\binom{n}{r}$	
expand $(x+y)^n$ for small positive integers $n$ .	

<b>Unit 1 Topic 4 – Exponential Functions 1</b>	
<b>Indices and the Index Laws (2 hours)</b>	
recall indices (including negative and fractional indices) and the index laws	A3-10 Index laws 1-5 A5-2 Index laws 6-10
convert radicals to and from fractional indices	A5-2 Index laws 6-10
understand and use scientific notation	N3-1 Scientific notation

<b>Unit 1 Topic 5 – Arithmetic and Geometric Sequences and Series 2</b>	
<b>Geometric Sequences (6 hours)</b>	
recognise and use the recursive definition of a geometric sequence: $t_{n+1}=rt_n$	A6-3 Geometric sequences
use the formula $t_n=t_1r^{(n-1)}$ for the general term of a geometric sequence and recognise its exponential nature	
understand the limiting behaviour as $n \rightarrow \infty$ of the terms $t_n$ in a geometric sequence and its dependence on the value of the common ratio $r$	
establish and use the formula $S_n=t_1(r^n-1)/(r-1)$ for the sum of the first $n$ terms of a geometric sequence	
establish and use the formula $S_\infty=t_1/(1-r)$ , $ r <1$ for the sum to infinity of a geometric progression	
use geometric sequences in contexts involving geometric growth or decay, including compound interest and annuities.	

## Unit 2 Topic 1 – Exponential Functions 2

### Introduction to Exponential Functions (6 hours)

recognise and determine the qualitative features of the graph of $y=a^x$ ( $a>0$ ), including asymptotes, and of its translations ( $y=a^x+b$ and $y=a^{x+c}$ )	A5-4 Exponential functions and logs
recognise and determine the features of the graphs of $y=b.a^x$ and $y=a^{kx}(k\neq 0)$	
identify contexts suitable for modelling by exponential functions and use models to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis	
solve equations involving exponential functions with and without technology.	

## Unit 2 Topic 2 – Exponential Functions 2

### The Logarithmic Function 1 (4 hours)

define logarithms as indices: $a^x=b$ is equivalent to $x=\log_a(b)$	A5-4 Exponential functions and logs
recognise the inverse relationship between logarithms and exponentials: $y=a^x$ is equivalent to $x=\log_a(y)$	
solve equations involving indices with and without technology.	

## Unit 2 Topic 3 – Functions and Graphs

### Circular Measure and Radian Measure (2 hours)

define and use radian measure and understand its relationship with degree measure	M6-1 Radians
calculate lengths of arcs and areas of sectors in circles.	

### Introduction to Trigonometric Functions (7 hours)

understand the unit circle definition of $\cos(\theta)$ , $\sin(\theta)$ and $\tan(\theta)$ and periodicity using radians	M5-1 Unit circle
recall the exact values of $\sin(\theta)$ , $\cos(\theta)$ and $\tan(\theta)$ at integer multiples of $\pi/6$ and $\pi/4$	M6-2 Exact trig values

sketch the graphs of $y=\sin(x)$ , $y=\cos(x)$ , and $y=\tan(x)$ on extended domains	A5-11 Trigonometric functions
investigate the effect of the parameters $A, B, C$ and $D$ on the graphs of $y=Asin(B(x+C))+D$ , $y=Acos(B(x+C))+D$ with and without technology	
sketch the graphs of $y=Asin(B(x+C))+D$ , $y=Acos(B(x+C))+D$ with and without technology	
identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis	
solve equations involving trigonometric functions with and without technology; including use of the Pythagorean identity $\sin^2(A)+\cos^2(A)=1$ .	A6-13 Trigonometric equations

## Unit 2 Topic 4 – Introduction to Differential Calculus

<b>Rates of Change and the Concept of Derivative (8 hours)</b>	
explore average and instantaneous rate of change in a variety of practical contexts	C6-1 Velocity graphically
use a numerical technique to estimate a limit or an average rate of change	C6-2 Velocity algebraically
examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \rightarrow 0$ as an informal introduction to the concept of a limit	
differentiate simple power functions and polynomial functions from first principles	
interpret the derivative as the instantaneous rate of change	
interpret the derivative as the gradient of a tangent line of the graph of $y=f(x)$ .	
<b>Properties and Computation of Derivatives (7 hours)</b>	
examine examples of variable rates of change of non-linear functions	C6-1 Velocity graphically
establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for positive integers	C6-2 Velocity algebraically C6-3 Velocity by rule
understand the concept of the derivative as a function	C6-2 Velocity algebraically
recognise and use properties of the derivative $\frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$	C6-3 Velocity by rule
calculate derivatives of power and polynomial functions.	
<b>Applications of Derivatives (10 hours)</b>	
determine instantaneous rates of change	C6-1 Velocity graphically

determine the gradient of a tangent and the equation of the tangent	C6-2 Velocity algebraically C6-3 Velocity by rule
construct and interpret displacement-time graphs, with velocity as the slope of the tangent	
sketch curves associated with power functions and polynomials up to and including degree 4; find stationary points and local and global maxima and minima with and without technology; and examine behaviour as $x \rightarrow \infty$ and $x \rightarrow -\infty$	C6-13 Graph sketchin
identify contexts suitable for modeling optimisation problems involving polynomials up to and including degree 4 and power functions on finite interval domains, and use models to solve practical problems with and without technology; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.	C6-14 Optimisation

## Unit 2 Topic 5 – Further Differentiation and Applications 1

### Differentiation Rules (6 hours)

understand and apply the product rule and quotient rule for power and polynomial functions	C6-6 Chain, product and quotient rules
understand the notion of composition of power and polynomial functions and use the chain rule for determining the derivatives of composite functions	
select and apply the product rule, quotient rule and chain rule to differentiate power and polynomial functions; express derivative in simplest and factorised form.	

## Unit 2 Topic 6 – Discrete Random Variables 1

### General Discrete Random Variables (5 hours)

understand the concepts of a discrete random variable and its associated probability function, and its use in modelling data	P1-1 Probability P6-3 Probability distributions and expected values S4-1 Quantiles and spread
use relative frequencies obtained from data to determine point estimates of probabilities associated with a discrete random variable	
recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes	
examine simple examples of non-uniform discrete random variables	
recognise the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases	
recognise the variance and standard deviation of a discrete random variable as a measure of spread, and evaluate these in simple cases	
use discrete random variables and associated probabilities to solve practical problems.	

## Unit 3 Topic 1 – The Logarithmic Function 2

### Logarithmic Laws and Logarithmic Functions (8 hours)

establish and use logarithmic laws and definitions	A5-13 Logs
interpret and use logarithmic scales such as decibels in acoustics, the Richter scale for earthquake magnitude, octaves in music, pH in chemistry	
solve equations involving indices with and without technology	
recognise the qualitative features of the graph of $y=\log_a(x)$ ( $a>1$ ), including asymptotes, and of its translations $y=\log_a(x)+b$ and $y=\log_a(x+c)$	
solve equations involving logarithmic functions with and without technology	
identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.	

## Unit 3 Topic 2 – Further Differentiation and Applications 2

### Calculus of Exponential Functions (8 hours)

estimate the limit of  $(a^h - 1)/h$  as  $h \rightarrow 0$  using technology, for various values of  $a > 0$

recognise that  $e$  is the unique number  $a$  for which the above limit is 1

define the exponential function  $e^x$

establish and use the formula formula  $\frac{d}{dx}(e^x) = e^x$  and  $\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$

identify contexts suitable for mathematical modelling by exponential functions and their derivatives and use the model to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.

C6-8 Mastering differentiation

### Calculus of Logarithmic Functions (8 hours)

define the natural logarithm  $\ln(x) = \log_e(x)$

recognise and use the inverse relationship of the functions  $y = e^x$  and  $y = \ln(x)$

establish and use the formulas formula  $\frac{d}{dx}(\ln(x)) = 1/x$  and  $\frac{d}{dx}(\ln f(x)) = f'(x)/f(x)$

use logarithmic functions and their derivatives to solve practical problems.

C6-8 Mastering differentiation

### Calculus of Trigonometric Functions (8 hours)

establish the formulas  $\frac{d}{dx}\sin(x) = \cos(x)$ , and  $\frac{d}{dx}\cos(x) = -\sin(x)$  by numerical estimations of the limits and informal proofs based on geometric constructions

identify contexts suitable for modelling by trigonometric functions and their derivatives and use the model to solve practical problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis

use trigonometric functions and their derivatives to solve practical problems; including trigonometric functions of the form  $y = \sin(f(x))$  and  $y = \cos(f(x))$ .

C6-8 Mastering differentiation

### Differentiation Rules (5 hours)

select and apply the product rule, quotient rule and chain rule to differentiate functions; express derivatives in simplest and factorised form.

C6-8 Mastering differentiation

<b>Unit 3 Topic 3 – Integrals</b>	
<b>Anti-differentiation (9 hours)</b>	
recognise anti-differentiation as the reverse of differentiation	C6-9 Differential Equations C6-10 Integration C6-11 Pert
use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals	
establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$	
establish and use the formula $\int e^x dx = e^x + c$	
establish and use the formulas $\int \frac{1}{x} dx = \ln(x) + c$ , for $x > 0$ and $\int \frac{1}{(ax+b)} dx = \frac{1}{a} \ln(ax+b) + c$	
establish and use the formulas $\int \sin(x) dx = -\cos(x) + c$ and $\int \cos(x) dx = \sin(x) + c$	
understand and use the formula for indefinite integrals of the form $\int (f(x)+g(x)) dx = \int f(x) dx + \int g(x) dx$	
determine indefinite integrals of the form $\int f(ax+b) dx$	
determine $f(x)$ , given $f'(x)$ and an initial condition $f(a)=b$	
determine the integral of a function using information about the derivative of the given function (integration by recognition)	
determine displacement given velocity in linear motion problems	
<b>Fundamental Theorem of Calculus and Definite Integrals (3 hours)</b>	
examine the area problem, and use sums of the form $\sum f(x_i) \delta x_i$ , to estimate the area under the curve $y=f(x)$	C6-10 Integration
use the trapezoidal rule for the approximation of the value of a definite integral numerically	
interpret the definite integral $\int_a^b f(x) dx$ as area under the curve $y=f(x)$ if $f(x) > 0$	
recognise the definite integral $\int_a^b f(x) dx$ as a limit of sums of the form $\sum_i f(x_i) \delta x_i$	
understand the formula $\int_a^b f(x) dx = F(b) - F(a)$ and use it to calculate definite integrals	
<b>Applications of Integration (6 hours)</b>	
calculate the area under a curve	C6-9 Differential Equations C6-10 Integration
calculate total change by integrating instantaneous or marginal rate of change	
calculate the area between curves with and without technology	
determine displacements given acceleration and initial values of displacement and velocity	

## Unit 4 Topic 1 – Further Differentiation and Applications 3

### The Second Derivative and Applications of Differentiation (9 hours)

understand the concept of the second derivative as the rate of change of the first derivative function	C6-12 Higher-order derivatives
recognise acceleration as the second derivative of displacement position with respect to time	
understand the concepts of concavity and points of inflection and their relationship with the second derivative	
understand and use the second derivative test for finding local maxima and minima	
sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection	C6-13 Graph sketching
solve optimisation problems from a wide variety of fields using first and second derivatives, where the function to be optimised is both given and developed.	C6-14 Optimisation

## Unit 4 Topic 2 – Trigonometric Functions 2

### Cosine and Sine Rules (9 hours)

recall sine, cosine and tangent as ratios of side lengths in right-angled triangles	M3-2 Trigonometry
understand the unit circle definition of $\cos(\theta)$ , $\sin(\theta)$ and $\tan(\theta)$ and periodicity using degrees and radians	M5-1 Unit circle
establish and use the sine (ambiguous case is required) and cosine rules and the formula $\text{area} = \frac{1}{2}bc \sin(A)$ for the area of a triangle	M5-2 Solving triangles
construct mathematical models using the sine and cosine rules in two- and three-dimensional contexts (including bearings in two-dimensional context) and use the model to solve problems; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis	

## Unit 4 Topic 3 – Discrete Random Variables 2

### Bernoulli Distributions (3 hours)

use a Bernoulli random variable as a model for two-outcome situations

identify contexts suitable for modelling by Bernoulli random variables

recognise and determine the mean  $p$  and variance  $p(1-p)$  of the Bernoulli distribution with parameter  $p$

use Bernoulli random variables and associated probabilities to model data and solve practical problems.

P6-4 Discrete random variables

### Binomial Distributions (5 hours)

understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in  $n$  independent Bernoulli trials, with the same probability of success  $p$  in each trial

identify contexts suitable for modelling by binomial random variables

determine and use the probabilities  $P(X=r) = \binom{n}{r} p^r(1-p)^{n-r}$  associated with the binomial distribution with parameters  $n$  and  $p$

calculate the mean  $np$  and variance  $np(1-p)$  of a binomial distribution using technology and algebraic methods

identify contexts suitable to model binomial distributions and associated probabilities to solve practical problems, including the language of 'at most' and 'at least'.

P6-4 Discrete random variables

## Unit 4 Topic 4 – Continuous Random Variables and the Normal Distribution

### General Continuous Random Variables (6 hours)

use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random variable

understand the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts

calculate the expected value, variance and standard deviation of a continuous random variable in simple cases

understand standardised normal variables (z-values, z-scores) and use these to compare samples

P6-5 Continuous random variables

<b>Normal Distribution (6 hours)</b>	
identify contexts, such as naturally occurring variations, that are suitable for modelling by normal random variables	P6-6 Normal distributions
recognise features of the graph of the probability density function of the normal distribution with mean $\mu$ and standard deviation $\sigma$ and the use of the standard normal distribution	
calculate probabilities and quantiles associated with a given normal distribution using technology and use these to solve practical problems.	

## Unit 4 Topic 5 – Interval Estimates for Proportions

<b>Random Sampling (3 hours)</b>	
understand the concept of a random sample	S2-1 Data collection
discuss sources of bias in samples, and procedures to ensure randomness	
investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli, using graphical displays of real and simulated data	P6-3 Probability distributions and expected values
<b>Sample Proportions (6 hours)</b>	
understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean $p$ and standard deviation $\sqrt{p(1-p)/n}$ of the sample proportion $\hat{p}$	P6-6 Confidence intervals for proportions
consider the approximate normality of the distribution of $\hat{p}$ for large samples	
simulate repeated random sampling, for a variety of values of $p$ and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{\hat{p}(1-\hat{p})/n}}$ where the closeness of the approximation depends on both $n$ and $p$	

<b>Confidence Intervals for Proportions (8 hours)</b>	
understand the concept of an interval estimate for a parameter associated with a random variable	P6-6 Confidence intervals for proportions
use the approximate confidence interval $\left( \hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$ as an interval estimate for $p$ , where $z$ is the appropriate quantile for the standard normal distribution	
define the approximate margin of error $E = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ and understand the trade-off between margin of error and level of confidence	
use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain $p$	