M1 Maths

ZOOMERS

Zoomers are open-ended tasks which fast students can engage with while other students are finishing work. Each task starts easy and gets harder. See how far you can go.

You might like to keep your work on the zoomers in a particular place so you can show it off to your teacher later. Your teacher might even give credit for it. But even if he or she doesn't, the thinking you do will greatly improve your mathematical prowess.

Your teacher might suggest a particular zoomer for you to work on, or you may be able to just choose your own.

10 20 30 40 50 60 70 80 90

For a Word version, click **here**

ZOOMER 1 (Levels 1 to 5)

Find the next terms in as many of these patterns as you can:

a) 11, 13, 15, 17, 19, …., ….

b) 37, 36, 34, 31, 27, 22, …., ….

c) 3, 6, 12, 24, 48, 96, …., ….

d) 1, 1, 2, 3, 5, 8, 13, 21, …., ….

e) 1, 4, 9, 16, 25, 36, 49, …., ….

f) 2, 7, 4, 9, 6, 11, 8, …., ….

g) 1, 3, 7, 15, 31, 63, …., ….

h) 1, 8, 27, 64, 125, …., ….

i) G, D, A, X, U, R, O, …., ….

j) O, T, T, F, F, S, S, E, N, …., ….

k) 1, 11, 21, 1211, 111221, 312211, 13112221, 1113213211, …., ….

l) I had a dream about dangerous reptiles eating a missionary. A bucket of unwashed turnips dangled above nine goannas eagerly ripping off uncooked skin. …….. …….. …….. ……..

[In this last one you need to find the next four words. Hint 1: There are millions of right answers, but trillions of wrong ones. Hint 2: Someone who doesn't speak English could do this just as easily as you.]

If you have someone else to work with, make up some patterns for each other to solve.

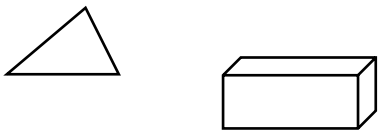
ZOOMER 2 (Levels 1 to 4)

Find 12 shapes, as different as possible, each with a perimeter of 12 cm. Sketch them and write on their measurements.

Ditto with 12 shapes each with an area of 12 cm^2.

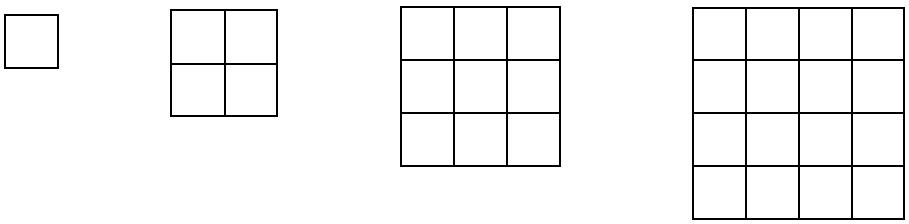
Ditto with 12 shapes each with a volume of 12 cm^3.

Ditto with 12 shapes each with a surface area of 12 cm^2.



ZOOMER 3 (Levels 1 to 6)

How many squares in these figures? Note that the second figure has 5, not all the same size.



How many squares on a chess board? Can you find a formula for the number of squares in such a figure and use it to find how many squares in a 1000 by 1000 figure?

How many rectangles in the figures above?

ZOOMER 4 (Levels 1 to 4)

13, 16, 19, 22, 25, ……

Find the 10^{th} number, the 40^{th} number and the 1000^{th} number.

Find a formula that allows you to work out any number quickly.

Find a formula for this sequence: 22, 18, 15, 13, 12, 13, 15, 18, 22, 27, ……..

ZOOMER 5 (Levels 1 to 4)

1st June 2019 was a Saturday.

What day is 1st June 2020?

How about 1st June 2040? 1st June 2099?

Can you find a way of finding the day of the week for 1st June 3142? You might need to look up rules for leap years.

You can do this quickly using an Excel spreadsheet. Format a cell to the date and enter any date you want. Then format another cell to Custom dddd and set it equal to your first cell.

This will work up to the year 9999. Can you find an algorithm for years beyond that?

ZOOMER 6 (Levels 1 to 6)

3 prisoners, A, B and C, have to be put into three cells. A could go in Cell 1, B in Cell 3 and C in Cell 2. Or A and C could go in Cell 1, B in Cell 2. Or they could all go in Cell 3. Or they could be arranged in other ways. How many ways altogether?

What if there were 4 prisoners and 4 cells?

5 and 5 cells?

6 and 6 cells?

Try to find a quick way of working these out so you can do 12 prisoners and 12 cells or 20 prisoners and 20 cells.

What if there are fewer prisoners than cells, e.g. 5 prisoners in 8 cells or 8 prisoners in 20 cells?

A new rule comes in that no two prisoners are to be put in the same cell. Repeat the calculations above taking this new rule into account.

ZOOMER 7 (Levels 1 to 3)

Write $^1/_7$ as a decimal (do 1 ÷ 7 on a calculator). Being a rational number, the decimal equivalent will either terminate or repeat. In this case it repeats because the denominator has prime factors other than 2 and 5. Starting with the first digit after the decimal point, write the sequence of digits that repeats.

Then do the same with $^2/_7$. What do you notice?

Without finding it, predict the repeating sequence for $^3/_7$. Then check with your calculator.

Check $^4/_7$ etc.

Can you find an overall pattern?

Try the same with other prime denominators.

ZOOMER 8 (Levels 1 to 5)

In a magic square, every row, every column and both diagonals add up to the same number.

This is a magic square.

Can you find others?

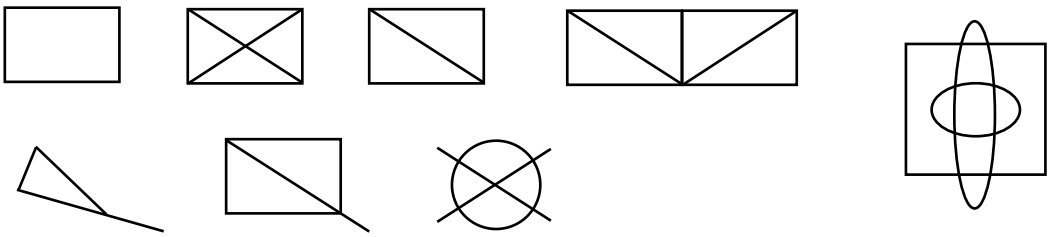
Can you find any bigger or smaller?

Can you find a magic oblong (a rectangle that isn't square)?

2	7	6
9	5	1
4	3	8

ZOOMER 9 (Levels 1 to 5)

Which of these shapes are traversable, i.e. can be drawn without taking your pencil off the paper and without going over any line twice.

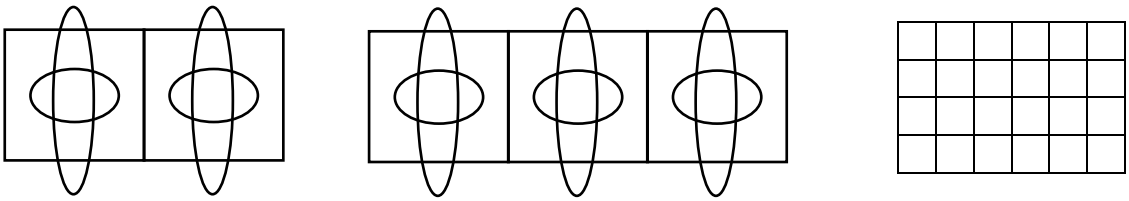


Pictures like these are sometimes called *graphs*. This is a slightly different meaning of the word from what you've met in Statistics. The word *graph* comes from the Latin word for *draw*, so *graph* really just means *drawing*.

On this type of graph, the points where lines or curves meet or end are called *nodes*. The line or curve connecting two nodes is called an arc. All arcs have two nodes, one at each end.

The order of a node is the number of arcs coming from it.

By looking at the orders of the nodes in the diagrams above, try to find a quick way to tell if a graph is traversable without trying it. Use your method on these.



A famous puzzle is *The Bridges of Konigsberg*. Konigsberg was a town in Prussia (now Kaliningrad in Russia). It had seven bridges over its rivers, something like the picture to the right. [Image from Wikipedia Commons]

The puzzle was to design a walk that took you over each bridge once and once only. Show whether this can or can't be done.

ZOOMER 10 (Levels 1 to 5)

If BEE is worth \$12, CAT \$24, DOG \$26, and FISH \$42, what is CROCODILE worth?

Can you find any $100 animals?

Hint, one of them tastes pretty good.

ZOOMER 11 (Levels 1 to 5)

4 people enter a race. All finish and there are no ties. In how many different orders could they finish?

What if there were a different number of people?

ZOOMER 12 (Levels 1 to 5)

Sometimes in maths it is convenient to handle two or more numbers together as a single unit. For instance, if Australia won 11 gold 16 silver and 9 bronze medals in the Olympics, this could be written $\begin{pmatrix}11\\16\\9\end{pmatrix}$. This set of numbers is called a vector.

The medal vector for England might be $\begin{pmatrix}7\\19\\16\end{pmatrix}$.

Who did better? Well, England won more medals – 40 compared to 36 for Australia, but Australia won more golds, so it depends on how you look at it. Often we decide by giving say 3 points for a gold, 2 for a silver and 1 for a bronze. We could write this as a points vector like this $\begin{pmatrix}3\\2\\1\end{pmatrix}$. We can then multiply the medals vector for Australia by the points vector, like this:

$\begin{pmatrix}11\\16\\9\end{pmatrix}.\begin{pmatrix}3\\2\\1\end{pmatrix} = 74$. This product is called the dot product of the two vectors because we use a dot instead of a normal multiplication sign.

Work out how to calculate a dot product by trying methods on the product above until you get the answer 74. The methods should be fairly obvious if you think about what is being worked out.

Do the same for England to see who did best.

Many people consider a gold to be worth a lot more than a silver and so prefer to use $\begin{pmatrix}4\\2\\1\end{pmatrix}$. Redo the calculations to find which country would do better in that case?

Find medal vectors for the two countries that would make them do equally well using the $\begin{pmatrix}4\\2\\1\end{pmatrix}$ points vector.

Vectors are commonly used to describe a movement (or displacement). For instance a movement to a point 20 m east of and 12 m north of the starting position could be described with the vector $\begin{pmatrix}20\\12\end{pmatrix}$.

What would be the vector for movement to a point 4 m east and 9 m north of the starting point?

If we use positive numbers for movements to the east and north, we can use negative numbers for movements to the west and south. What would be the vectors for a movement to a point 18 m east and 13 m south of the starting point?

How about to a point 5 m west and 7.2 m south of the starting point?

If you do Specialist Maths or Physics in Years 11 and 12, you will do quite a bit with vectors.

ZOOMER 13 (Levels 1 to 5)

You can draw 2 different diagonals in a square, 5 in a regular pentagon.

How many different diagonals can you draw in a regular hexagon?

Make a table for different regular polygons, then try to find a pattern that will let you work it out for a regular centagon (100 sides).