M1 Maths

Learning Maths – Why and How

Why Learn Maths?

Everyday Life

Much of what we do in everyday life involves quantities and quantitative reasoning. Money is an obvious example. If you see a pair of shoes in a sale for $90 and decide to buy them, it's not sufficient to have *some* money. You must have at least $90.

'Ah, but . . .' you say. You can get them on credit. This is true, but a credit card is not a bottomless pit. You need to have sufficient credit available.

Also, it's good if you can take into account more than just whether there's enough money for that pair of shoes. You might need to buy some food with that last bit of credit.

Using credit to buy shoes is often a sign of not being able to deal with one's finances well. If you normally owe $6000 on your credit card, you may be paying $1000 a year in interest and fees. If you waited until you had the money for things you buy, you could save that $1000 a year and in the long run, be able to buy a lot more with your money. A lot of people live their lives in unnecessary poverty because they do not understand the ins and outs of money, credit, interest etc. Banks do very well out of those people.

There are other financial matters where having a bit of mathematical ability is vital. These include knowing if you are being paid right, budgeting, saving for a new car or holiday, investing etc.

Many people who start smoking wouldn't do so if they worked out what it would cost them in the long run. In Australia, a pack a day from the age of 15 to 55 will set you back around half a million dollars. That's definitely enough to make the difference between being well off and never having enough money to do the things you want to do. If your partner smokes too, that's around a million dollars between you.

Many jobs people need to do around the house require a certain amount of mathematical knowledge. Painting, putting up a fence, making a concrete path in the garden, buying some gravel for the drive all require reasonable competence with

measurement ideas – length, area, volume, units etc. and maybe a bit of Pythagoras and trigonometry if you build things and you want them to be the right shape.

Careers

Competence in maths is needed for most jobs and a high level of competence is needed for some. Many of the better paid jobs are ones which require a higher level of maths.

Some professional sports people might make more money than engineers, but very few people make money out of sport. Of course, we hear about the few who do. On the other hand, there are many thousands of engineers in the country, all with a good steady income that will last them throughout their working lives. As well as the engineers, there are electricians, scientists, medical professionals, economists, bankers etc., all making much more money than people doing jobs like shop assistant, cleaner, gardener, road worker etc., jobs that require a lower level of maths.

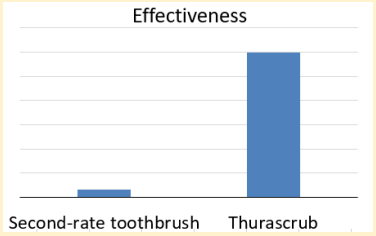
Even for jobs which aren't terribly mathematical, like making burgers in Macca's, the interviewer is generally interested in the applicant's mathematical achievement, not so much because being good at maths is essential for the job, but because being good at maths is an indication of (a) the applicant's intelligence and (b) the applicant's ability to work at something he or she may not particularly want to work at.

Critical Awareness

People try to convince us of many things through the media in order to get us to buy their product, vote for them or just to agree with their position on a controversial issue. Often their argument involves quantitative data and reasoning and the reasoning is often not valid. Often they know this, but they use it anyway because they know that a lot of people will not have sufficient mathematical skills and knowledge to see the flaws in their argument and so will believe them anyway.

The advert to the right is hypothetical and somewhat extreme. Hopefully, you can see the many flaws in the argument (see the Lead in to Module S3-3 for a list of some of them), but similar tactics are used a lot.

In a recent survey of leading dentists, it was found that 4 out of 5 recommend Thurascrub toothbrushes.



So, if you want healthy white teeth, throw out that second-rate toothbrush and get a Thurascrub. On sale at all good chemists and supermarkets.

Another example is the statement:

'An independent survey shows that people who drive BMWs are, on average, three times as rich as people drive Fords. So if you want to be rich, buy a BMW.'

Competence at mathematics allows us to see through spurious arguments and to not be fooled into making decisions which are in the interests of those who are persuading us rather than in our own interests.

There are many other ways that we can be taken advantage of if we aren't mathematically aware. A man turned up to a landscape supply place to buy a cubic metre of gravel. He knew his trailer measured 1.75 m in length, 1.32 m in width and 40 cm in depth, giving it a capacity of 0.924 m^3 if filled to the brim, but not heaped up in the middle.

After the supplier put the gravel in, the man thought it looked like less than a cubic metre, so he flattened it down and found that it didn't reach the rim of the trailer. Thus he had received less than 0.924 m^3. He argued this with the supplier who insisted that he had given him the right amount. The man offered to measure the trailer to prove his point and did so. The supplier then topped it up for him.

Most people just take it for granted that they are getting the right amount when they buy something like that and suppliers often take advantage of that fact.

Probability is a subject poorly understood by many people. No one who understands it would play on the pokies or casino games in the hope of making money in the long run. One might play very occasionally for a bit of fun, knowing that they will more likely lose money, but never as a long-term or habitual pursuit. In the long term they must lose. Absolutely must.

Some will claim that they win in the long run, but, when asked whether they keep records of their wins and loss, invariably they don't. They just remember the wins more because they are more unusual and because they celebrate them.

People buy lotto tickets choosing their numbers on the basis of numbers that have come up in recent draws. Of course, what has happened in past weeks will have absolutely no impact on which balls fall out of the machine this week.

A pub game involves a croupier shaking a couple of dozen dollar coins, one of which has a white spot on both sides, in a bag, arranging them into a stack without looking at them, then placing the stack on the bar. Players then make bets on whether, when the top coin is removed, the next coin will be heads up or tails up. If they bet $5 and are wrong, the croupier keeps their $5; if they are right, the croupier gives them their $5 back plus another $5. If the next coin has the spots, the croupier keeps all the bets.

Many players will wait for a string of 3 or 4 heads or 3 or 4 tails, then bet on the opposite, relying on the law of averages which says that the number of heads and tails must even out in the long run. Of course, whether the last few coins were heads or tails makes no difference to the next coin. In the long run, players will lose money to the croupier because of the coin with the spots.

How to Learn Maths

To be effective at doing maths, we need both knowledge and skills. Knowledge without skills makes us able to do things we have been specifically taught to do, but unable to do anything different. Skills without knowledge makes us capable of doing any maths, but at a pitifully slow rate.

It is therefore important that we devote time and effort to developing knowledge (by memorising and rehearsing facts and techniques) and developing skills (by solving problems of types that we haven't specifically learnt to solve, by investigating mathematical ideas we haven't met before etc.).

Traditional school maths programs tend to devote a lot more time to learning knowledge than to developing skills. Part of the reason for this is that the curriculum expects and requires the development of a lot of knowledge and this leaves little time for the skills. However, if more time is spent developing skills, then the student will more readily see the sense in many of the techniques that have to be learned and so will pick them up more quickly and remember them more easily. Many of the techniques required can in fact be worked out quite simply and quickly by a student with good skills, while a student with poor skills needs to memorise them parrot-fashioned. So giving more attention to skills and less to knowledge can result in the possession of just as much knowledge in the long run.

It is suggested, therefore, that plenty of time is devoted to practicing problem solving, investigating and mental arithmetic.

What makes one person better at maths than another?

There are 7 factors that contribute to being good at maths. They are listed below with their estimated percentage contribution. You have control over 6 of the 7.

1. Innate aptitude (20%)

How to get good	How not to get good
Be born with a brain wired for natural ability in maths	Be born with a brain not wired for natural ability in maths

Obviously, this is the one factor you don't have control over. Some people are lucky enough to be born with brains that are hard-wired for exceptional ability in maths. Some of the famous mathematicians of history like Archimedes and Isaac Newton were examples of this. Others, less fortunate, have brains wired in such a way as to make learning slower and more of a challenge.

Fortunately, though, the vast majority of people are in between these extremes and have fairly similar innate ability. Some might pick things up a bit quicker than others,

but nearly everyone can develop their mathematical ability at a good rate. In nearly all cases, it is the other 6 factors which make the most difference.

The author once taught a student who, based on past performance, was in the lowest level maths class in Year 9. However, his uncle, who was obviously very rich, had recently offered him a million dollars if he became a doctor. Although this meant doing well in the hardest level of maths in Years 11 and 12, he set his mind to it. He worked very hard while in my class, taking all the tasks seriously and going beyond the work that I set. I ran into him several years later. He was a qualified engineer – a career that required even more maths than medicine. I didn't ask about the deal with the million dollars, but hopefully, his uncle considered engineering sufficiently good.

2. Learned aptitude (past mental stimulation (learned intelligence), thinking skills, reaction to problem situations, confidence) (25%)

How to get good	How not to get good
Get plenty of mental stimulation that makes you think.	Stick to situations which are fairly routine, repetitive and mindless, like watching junk TV.
Take steps to develop your thinking skills: solve problems, learn strategies etc.	Assume that you don't need to think much or that you can't or don't need to improve your thinking skills.
Treat problems as challenges	Avoid problems so you don't have to do any hard thinking. Let someone else work things out.

From the time we are very young, mental stimulation develops our brain and our ability to think, understand and work things out. Admittedly, once we are in high school, we have no control over what happened to us when we were babies, but the mental stimulation we get then can still develop those skills. Challenging ourselves with getting to understand mathematical concepts and solving problems that require us to think hard provide this stimulation. We can get the stimulation we need by embracing opportunities to think hard rather than avoiding them as many people do because they require effort.

Making mental effort is like making physical effort: it can be off-putting if we're not into it, but it can be fun and rewarding if we are. Try to cross that threshold to where it is fun.

Confidence comes with becoming good at something (or even just becoming better than you were). Confidence brings the added bonus of enjoyment. There are two ways to enjoy life: one is to do what you like; the other is to like what you do.

3. View of learning (15%)

How to get good	How not to get good
View mathematical knowledge / ability as something cool	View mathematical knowledge /ability as nerdy and uncool
View effort at learning as cool	View effort at learning as nerdy and uncool
View learning as desirable, important and worth time and effort	View education as an imposed inconvenience to be side-stepped if possible to devote the time to more pleasant pursuits
View learning as the development of unified and coherent understanding by which all knowledge can be explained (analogous to the hierarchical structure of mathematical knowledge based on axioms)	View learning as understanding and retention of isolated facts with no need to know why they are so or to see connections between them
See things that don't make sense and paradoxes as something that needs to be resolved	Don't be interested in anything that doesn't make immediate sense
See a need to reconcile different views of a situation or different methods to see how they fit into the big picture and fit with each other	Don't be interested in more than one method or view
See knowledge as being built to last	See knowledge as something to maintain until the next test

4. Approach to learning experiences (15%)

How to get good	How not to get good
Use the experience for learning as far as possible	Just try to get the work finished and out of the way
Be prepared to make major effort	Attempt to get by with minimum effort
Organise and refine your mental map and make sure everything fits into it and makes sense	Remember things in isolation; don't worry about how they relate to other things
Develop and use effective memorisation techniques	Just hope to remember

Aim to remember permanently by ensuring that the idea follows obviously from other knowledge	Aim to remember for the next test

5. Thought intensity (15%)

How to get good	How not to get good
Focus on the maths, think intensely	Make half-hearted effort, thinking about something else at the same time, e.g. a casual conversation with the people around you, what's happening on the phone, a TV show, a worry
Develop persistence, determination, tenacity	If, at first, you don't succeed, give up – it's not worth it

6. Responsibility for learning (10%)

How to get good	How not to get good
Feel responsible for learning, to find out what needs to be learnt and to ensure that it is learnt	Feel that your learning is the teacher's responsibility and that, if the teacher doesn't do a good job, then you aren't to blame – it's the teacher's fault and that's just too bad
Take steps to find out how the material can be learnt, e.g. useful parts in the text book, other resources and how to use them	See that as the teacher's job
Take steps necessary to ensure learning is successful and persist until it is	Leave that for the teacher t worry about
See the teacher as a resource, someone who can guide and help you in your learning	See the teacher as someone who makes you do things you might not want to do.
Take opportunities to do more activities that will lead to learning if you have time	Try to get away with doing as little as possible – only work if not doing so will lead to trouble
Try to get as much as possible done now and keep up to date or ahead	Don't do things now if there will be time later: don't worry about starting until there is only just enough time left

7. Time spent learning

1-6 will determine the rate of learning. Multiplying this by the time spent learning will yield the amount of learning achieved.