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Shifting the Emphasis from Memorising to Thinking

Presentation at the 2023 QAMT Annual Conference *David Ilsley (d.ilsley@gmail.com)*

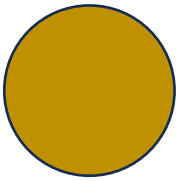
Peter and Paul

Consider two students, Peter and Paul, who are each given the problem below.

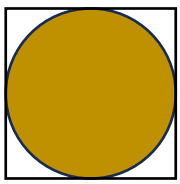
This is the dirt patch inside a small traffic roundabout.

Its radius is 5 m.

How many square metres of turf would be needed to turf it?



Peter looks at the problem, says it's a circle and says 'The formula is πr^2 . . . or is it $2\pi r$? . . . it's one of those . . . I always confuse those two'. After a moment's indecision, he decides to go with πr^2. He takes his calculator and comes up with 78.5 m^2.



Paul looks at the problem, says he doesn't know how to find the area of a circle, thinks for a moment, then says 'I can do squares, though.' He draws a square snuggly around the circle, points out that the square must be 10 m by 10 m, and so its area would be 100 m^2. After a little more thought, he says 'We would need less turf than that though because we don't have to turf the corners. We would probably need about 80 m^2.

The workshop participants were asked: 'Which student had the most potential as a mathematician?' The consensus was that it was Paul. Though he didn't know the required formula, he was able to think out an approach to the problem. Most agreed that the thinking ability that Paul showed is more important than the formula knowledge that Peter had.

And yet, the tendency in schools is to spend a lot more time learning and reinforcing facts, formulae and procedures than developing thinking skills. A major part of the reason for this is that, for many students, just learning the facts, formulae and procedures required by the curriculum takes more time than is available. As these

tend to be the main focus of assessment, the development of thinking skills often tends to be squeezed out.

The ironic thing is that, if students are good thinkers, they will be able to work out a lot of the formulae and procedures themselves, thus will learn them more quickly and forget them less readily. In the long run, they would probably learn more maths in the time available if thinking skills were given a larger portion of the available time. The trouble is, though, that while that applies in the long term (a few years), people worry that it might not be true over the shorter term (a year or less). A teacher or program that spends more time on thinking and less on memorisation might worry about producing poorer assessment results in the short term and this might be considered to reflect negatively on them and their teaching approach.

Jake and Melissa

Peter and Paul were fictitious, but Jake and Melissa are real – they are my children.

There is a tendency for students to develop the view that there is a proper way to do everything in maths and that, if you don't know that way, you can't do the maths. Once, when broken down in the family car and waiting for rescue, I asked Jake (who was then in Year 2) 'What's 1000 take 60?' Some call this child abuse, but hey! I'm a maths teacher. Jake looked pensive, then started to count on his fingers, mumbling quietly, then said '940'. He had counted down in 10s.

Melissa (then in Year 4) wasn't in the car, but when she came back, I asked her the same question. She thought for a moment and said, 'I need some paper.' I provided some paper and a pencil. She wrote

$$\begin{array}{r} 1000 \\ -\ \ 60 \end{array}$$

She took 0 from 0 to get 0. Then said, 'I can't take 6 from 0'. I said, 'No. Can you do it some other way?' She thought for a moment and said, 'No.'

Both are of comparable intelligence. I believe the difference was that Melissa had had two extra years to learn that there is a right way to answer any maths question and that, if you don't know it, you can't answer the question. This seems to be something that was inadvertently instilled into her through her maths instruction. It does seem that it is possible to teach students not to think and sometimes this happens.

Springwood SHS

Another experience, however, tells me that the reverse can also be true – we can teach children to think and, in the process, make them better mathematicians.

In the 1990s I was working as a maths advisor for the South Coast Region of Education Queensland. The head of maths at Springwood SHS and I agreed that I would take one of her Year 8 classes for a year and teach, as far as possible, through

problem solving. I did the absolute minimum of explicit content teaching, instead developing their problem-solving skills through problems which didn't require specific factual or procedural knowledge and developing their content knowledge by giving them stepped questions designed to lead them to come up with the knowledge by themselves though logic. An example of this type of question aimed at developing the concepts of rate and proportion, is:

A team of 3 people can pack 42 boxes of peaches in an hour. How many boxes could 5 people pack in 8 hours?

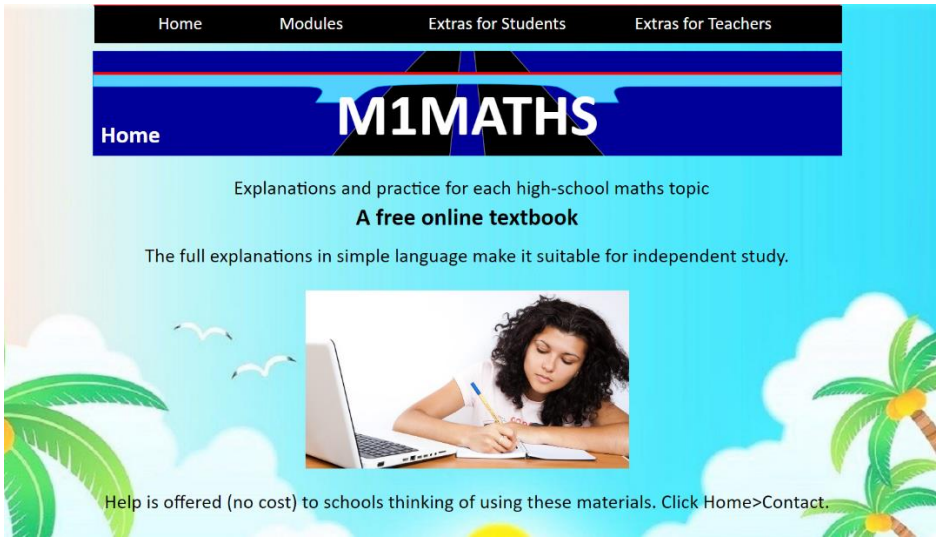
After the first semester, the head of maths was impressed with what the class was doing and suggested that the whole Year 8 cohort take the same approach for Semester 2. We did that.

The next year, I started teaching full-time at Beenleigh SHS and the leadership at Springwood changed, so the experiment didn't continue. But the students who we had experimented with then formed the Year 9 cohort. That year, as usual, Springwood put a few teams in the Maths Team Challenge held at Woodridge SHS. In the Years 9-10 Division, though there were teams from about 10 schools, Springwood teams got four of the top 5 places. This was despite spending Year 8 being explicitly taught next to no content knowledge.

All this suggests that shifting the emphasis from memorising facts, formulae and procedures to developing thinking and problem-solving skills can pay dividends and might well improve maths education outcomes, even in the relatively short term.

Resources

To make this shift, however, we need suitable resources. There are more than enough in existence. But I am going to mention a few from the M1Maths website (m1maths.com).



I will use this for a number of reasons: one is that the resources are available to everyone online and are totally free with no login and no ads; another is that the site has enough to provide a good program without going to several different places; another is that the resources are written so as to be accessible to averagely literate students even without teacher instruction. One more reason, which I should mention, is that it is my website. But, as I said, it is free, so I have no vested interest in it being used.

The site is quite extensive, but the following sections may be useful for the present purpose.

The 'Problem Solving' module contains around 300 problems, each assigned to a level. Level 1 is aimed at about Year 7, Level 6 to Maths Methods. Along with the problems, there are explanations of how to go about solving problems, how to use the various strategies etc.

The 'Investigations' module does the same thing with open-ended investigations. Doing a few of these is important for students to get a feel for maths as something that they can explore and develop themselves.

The regular knowledge modules, each have, in addition to explanations and practice questions, a set of problems related to the topic of the module.

Some modules (e.g. M1-4) have a 'Learning by Thinking' alternative to the explanation and practice section. These are designed to lead the students to the required knowledge by solving problems with a minimum of direct instruction – in the manner of the teaching at Springwood.

All these are available via the 'Main Menu' button.

Another 2500 problems can be found in the 'Maths Team Challenge' section under 'Extras for Teachers'. These are questions written between 1994 and the present by Stephen Broderick and Bob Nelder for the SE Queensland Maths Team Challenge competitions. There are divisions for Years 5-6, Years 7-8, Years 9-10 and Years 11-12.

Finally there are various problem-solving games elaborated under the 'Fun and Games' section, also under 'Extras for Teachers'. My favourite is the 'Group Problem Solving' game which tends to get even the less enthusiastic problem solvers thinking hard and sharing ideas in order to beat the other teams, especially if a small prize is at stake. Other problem solving-games in the 'Fun and Games' section include the 2 by 5 Challenge, the 12 Question Challenge, the Relay and the 4-Corner Quiz.

Assessment

Of course, for thinking to be taken seriously by the students, it has to be a major part of the regular assessment, just the same as content knowledge does. Suggestions and examples of assessment items can be found in the 'Level Tests' section under 'Extras for Teachers'.

A Bad Memory Can Be a Good Thing

One final thought. A bad memory can be a positive in learning maths. The following is the result of a statistical investigation I carried out with a sample size of one, me. I have always had a bad memory. As a result, when I was at school, I was usually around the bottom of the class in subjects like history and languages, where memory was important. Yet I was always around the top of the class in Maths. In retrospect, I believe this was because I couldn't rely on remembering things that didn't make total sense. I knew I had to be able to think everything out again from basics in the very likely event of forgetting a procedure. I naturally learnt by thinking rather than by memorisation.

The PowerPoint used in the presentation is available on m1maths.com under 'Extras for Teachers' then 'PD'. Go to the last item.