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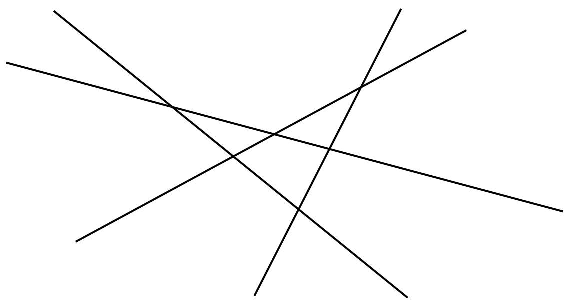
# Angles formed by intersecting and parallel lines
# Two alternative approaches

## 1. The theorem approach

Deductive geometry usually begins with theorems involving vertically opposite angles, co-interior angles, angles in a triangle etc and the use of these to determine the magnitude of angles, given the magnitudes of other angles, on geometric diagrams.

The theorems can be taught directly to the students, but the following activity is an alternative way of getting students to discover these relationships for themselves.

First ask the students to draw four straight lines on a sheet of paper so that each line crosses all the others, but with the intersections not too close together.



This produces 24 angles. Ask the students to measure each of the 24 angles and to write the number of degrees in the angle.

When they have done this, ask whether anyone noticed anything which allowed them to predict some of the angles before they measured them. Many will have noticed that vertically opposite angles are always the same (though they probably won't use the term 'vertically opposite').

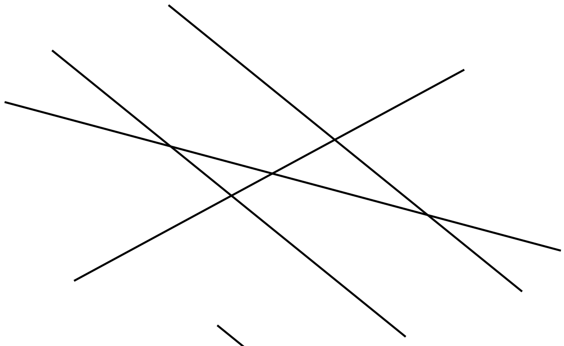
Then ask them to draw up another set of four lines and to find all the 24 angles, but this time ask them to see how few angles they can measure in doing so. Allow students to let the others know if they find a way of doing it with less measures than anyone else has done. Most students will need to use at least three or four sets of lines. These can be pre-made and photocopied.

Eventually most should be able to do the job with only three measurements. Get some of the students to show the class the methods they used.

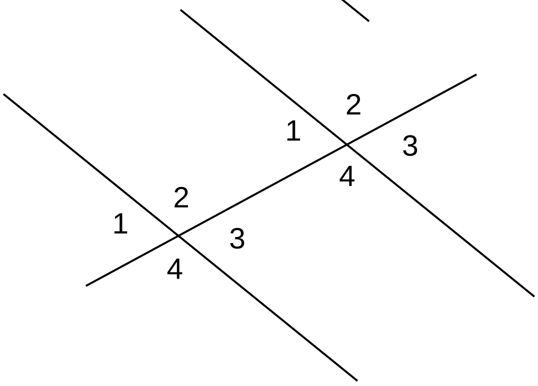
Five ideas should come out:

- supplementary angles or T angles;
- angles at a point or Y angles;
- vertically opposite angles or X angles;
- angles in a triangle or D angles; and
- angles in a quadrilateral or O angles.

Then (or a few lessons later) repeat the exercise, but this time make two of the lines parallel. (These can be drawn using the two sides of a ruler.)



There will now be 20 angles. This will add the ideas related to parallel lines and a transversal to the students' repertoire. But they are unlikely to identify separate rules about alternate, corresponding and co-interior angles. They will probably just note that where a line crosses two parallel lines, it makes the same four angles in the same position at each intersection.

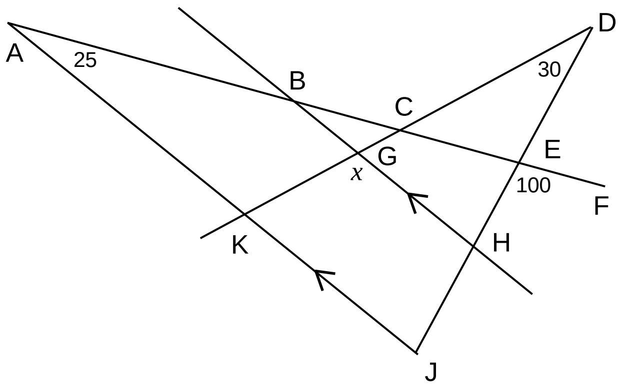



This is the H rule to complete the set TYXDOH. This mnemonic isn't too difficult to remember as a checklist for later problems.

## 2. The bearings approach

The theorems above are the traditional way of solving problems like the following:

Find the magnitude of angle $x$ in the diagram below.




The solution might look something like this:
∠DEC = ∠FEH = 100° (X)
∠DCE = 180° – ∠CDE – ∠DEC = 50° (D)
∠GCB = ∠DCE = 50° (X)
∠CBG = ∠BAK = 25° (H)
∠BGC = 180° – ∠CBG – ∠GCB = 105° (D)
$x$ = ∠BGC = 105° (X)

But there is another approach based on the orientation of the lines in the diagram. We can describe the orientations as bearings.

Consider line JA to have a bearing of 0°. AJ is then 180°. Turning the diagram round so that JA is vertical can help.

AF is 25° less than AJ, ie. 155°.
DJ is 100° more than AF, ie. 255°.
DK is 30° more than DJ ie. 285°.
GH is parallel to AJ and therefore 180°
$x$ is the difference between DK and GH, ie. 105°.

This method is not a lot shorter, but it does have two advantages. Firstly it doesn't require the memorisation of any theorems. Secondly it requires and reinforces the concept of bearing, something used in senior maths and used more in real life than the geometric theorems.